

PERSPECTIVES ON BOSON SAMPLING

BY: KEITH R. MOTES

eQus

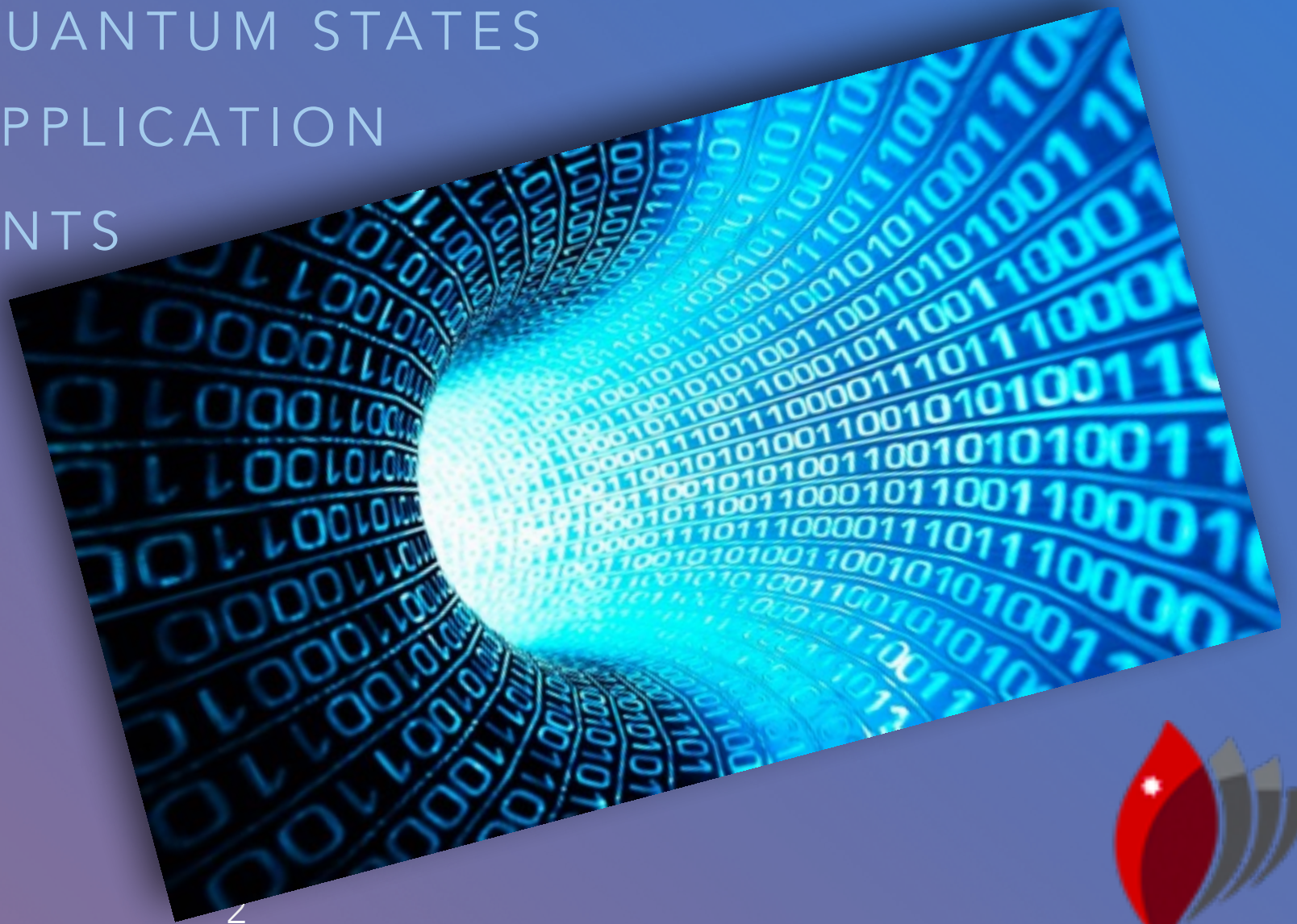
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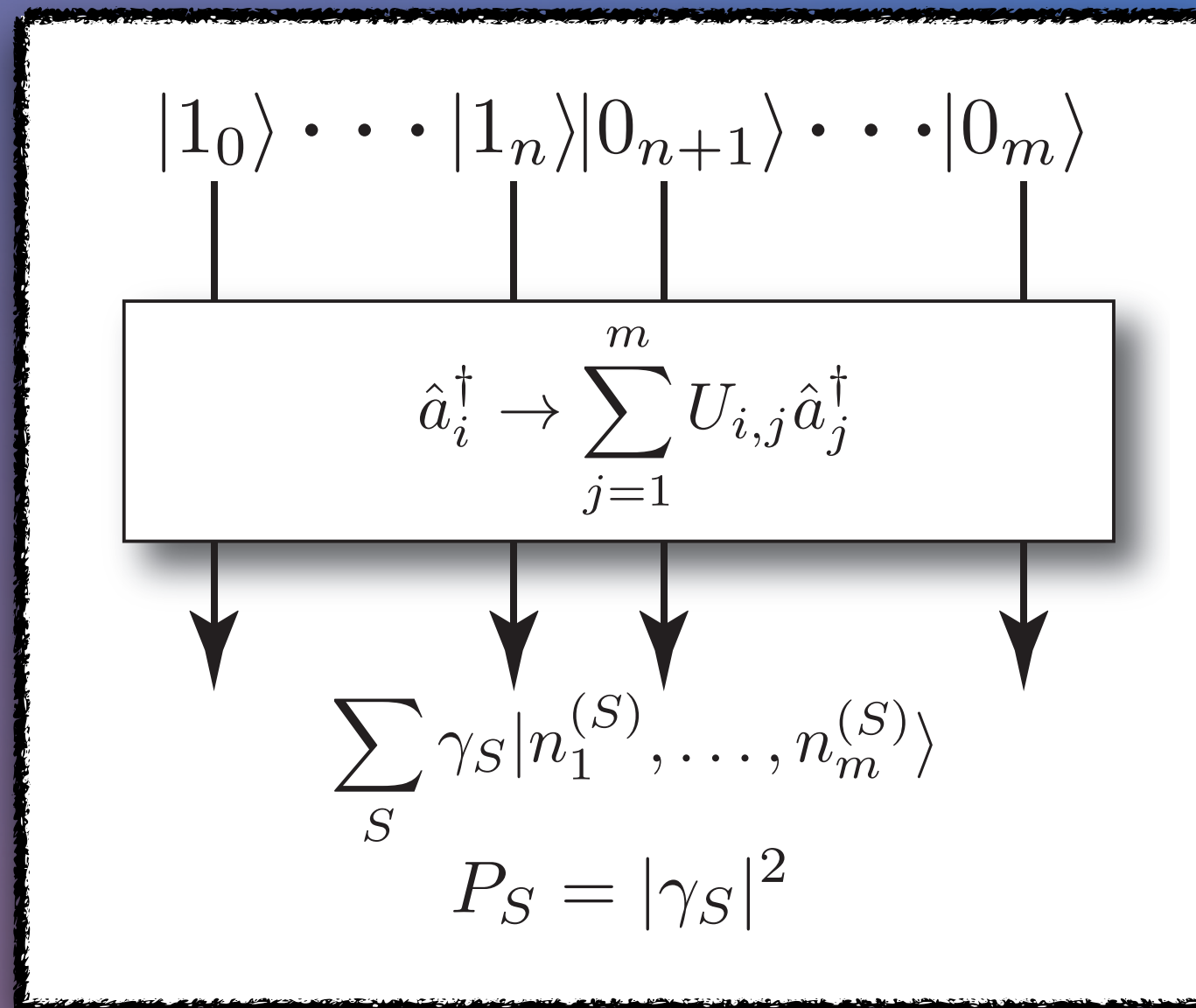


OUTLINE: BOSON SAMPLING

1. INTRODUCTION & MOTIVATION
2. SPDC SOURCES
3. BOSON-SAMPLING IN TIME WITH ERROR ANALYSIS
4. SAMPLING OTHER QUANTUM STATES
5. A METROLOGICAL APPLICATION
6. LATEST DEVELOPMENTS



BOSON-SAMPLING



$$|S| = \binom{n+m-1}{m}$$

$$\gamma_S \propto \text{Perm}(\hat{U})$$

Boson-sampling is
#P-hard

The signal is $P = |\text{Perm}(U)|^2$



BOSON-SAMPLING

$$\text{Perm}(A) = \sum_{\sigma \in S_n} \prod_{i=1}^n a_{i,\sigma(i)}$$

$$\text{Det}(A) = \sum_{\sigma \in S_n} \text{sgn}(\sigma) \prod_{i=1}^n a_{i,\sigma(i)}$$



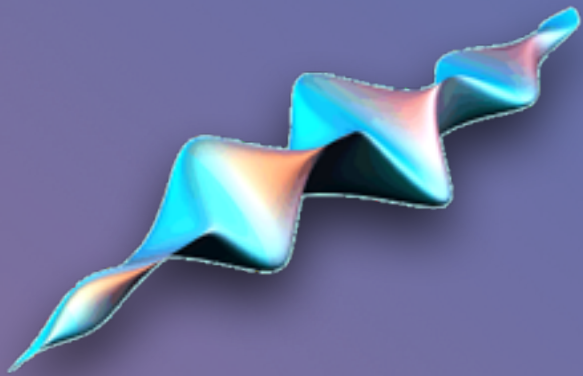
Ryser's Algorithm

$$O(2^n n^2)$$

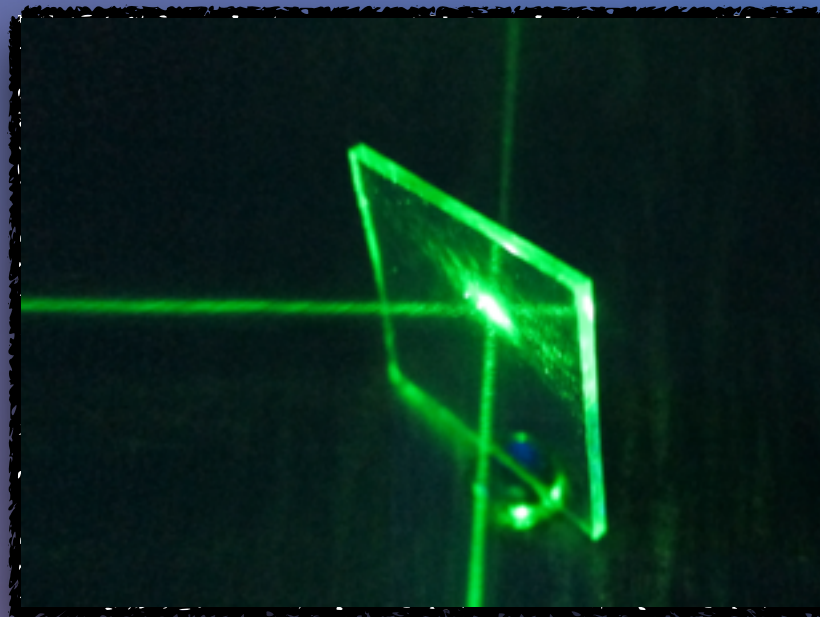


BUILDING A BOSON-SAMPLER

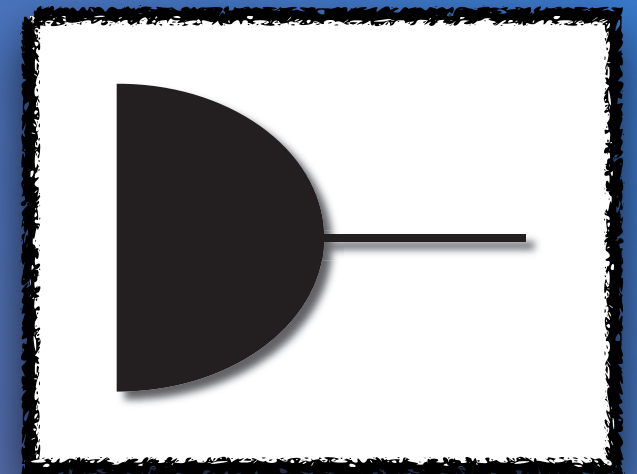
SOURCES:



EVOLUTION:



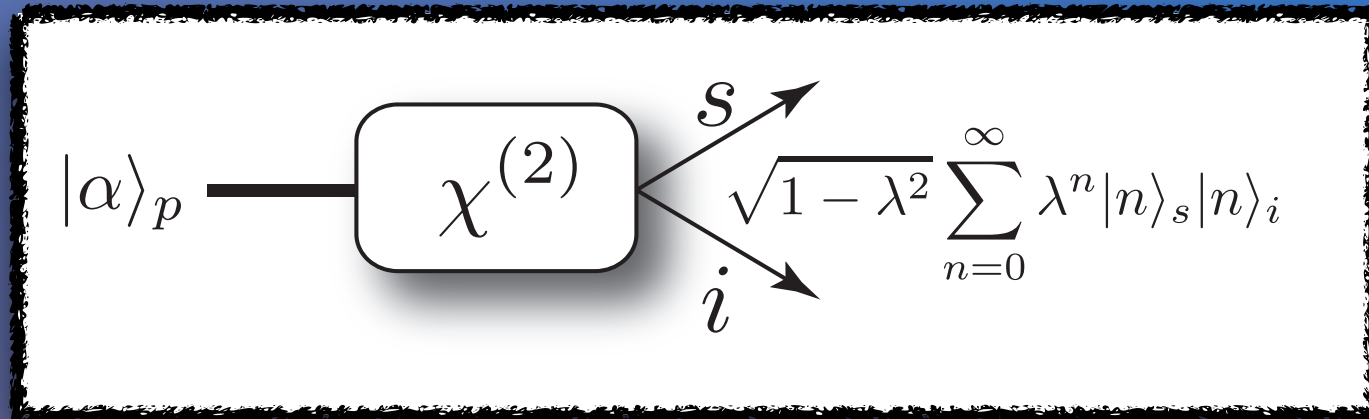
DETECTION:



SPDC SOURCES

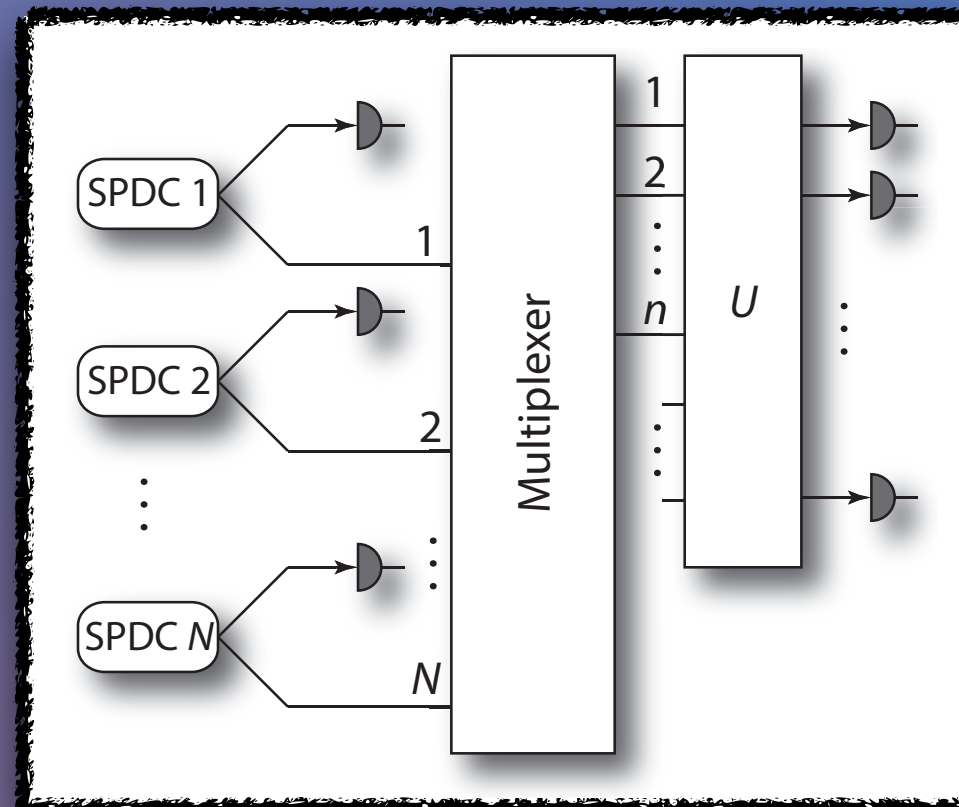
SPONTANEOUS PARAMETRIC DOWN CONVERSION (SPDC)
SOURCES FOR BOSON-SAMPLING

SPDC SOURCE:



$$P_D(t|s) = \binom{s}{t} \eta^t (1 - \eta)^{s-t}$$

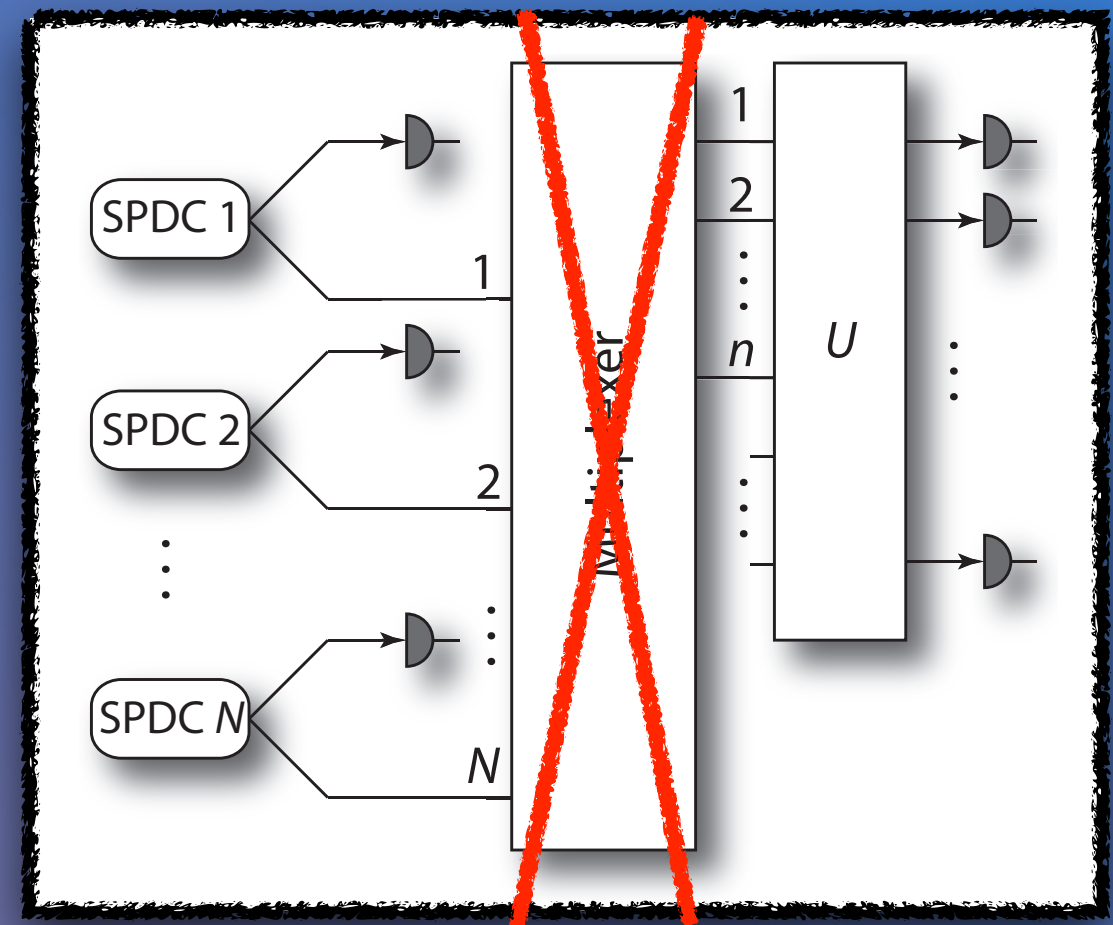
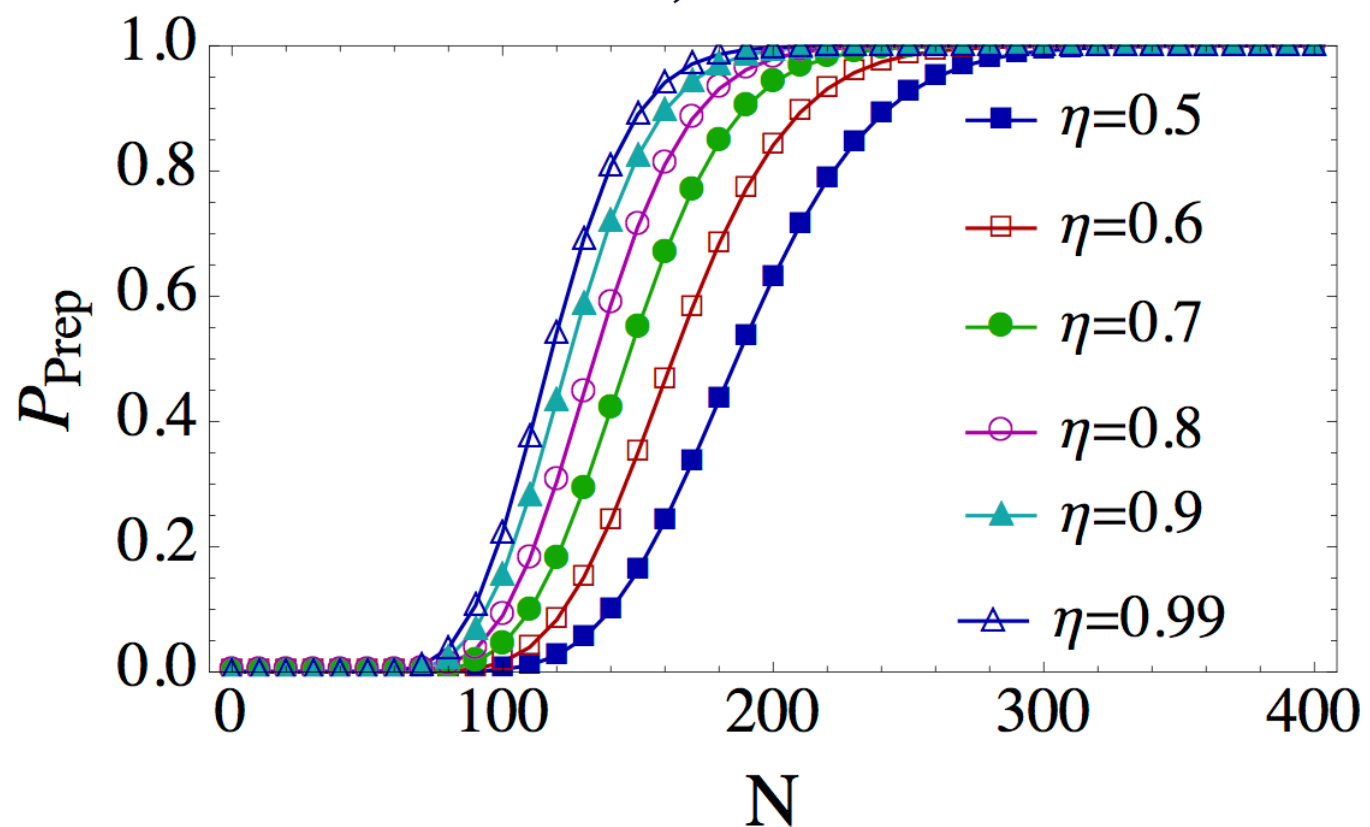
MODEL:



SOURCES

SPONTANEOUS PARAMETRIC DOWN CONVERSION (SPDC) SOURCES FOR BOSON-SAMPLING

$n=20, r=0.5$



LUND ET AL., PRL 113, 100502 (2014)



SCALABLE BOSON-SAMPLING WITH TIME- BIN ENCODING USING A LOOP BASED ARCHITECTURE

BY: KEITH R. MOTES, ALEXEI GILCHRIST,
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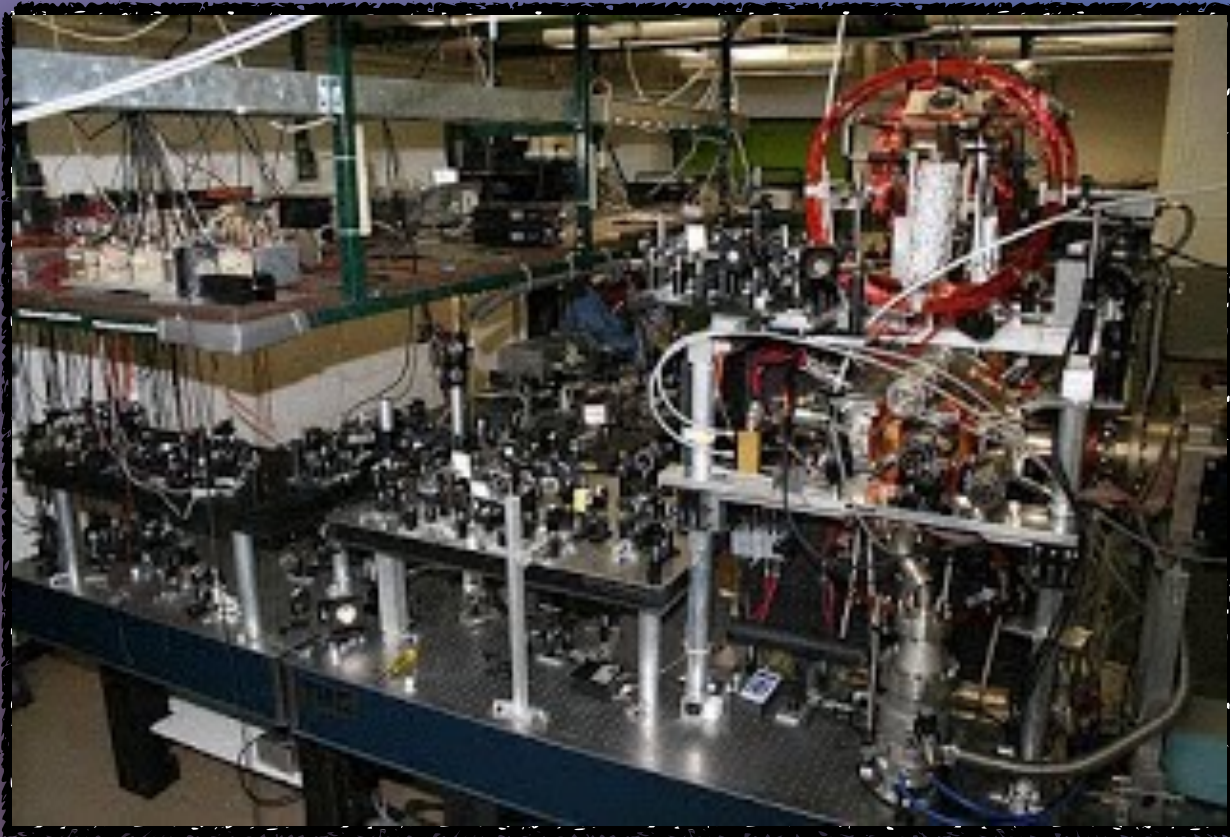
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SPACE VERSUS TIME

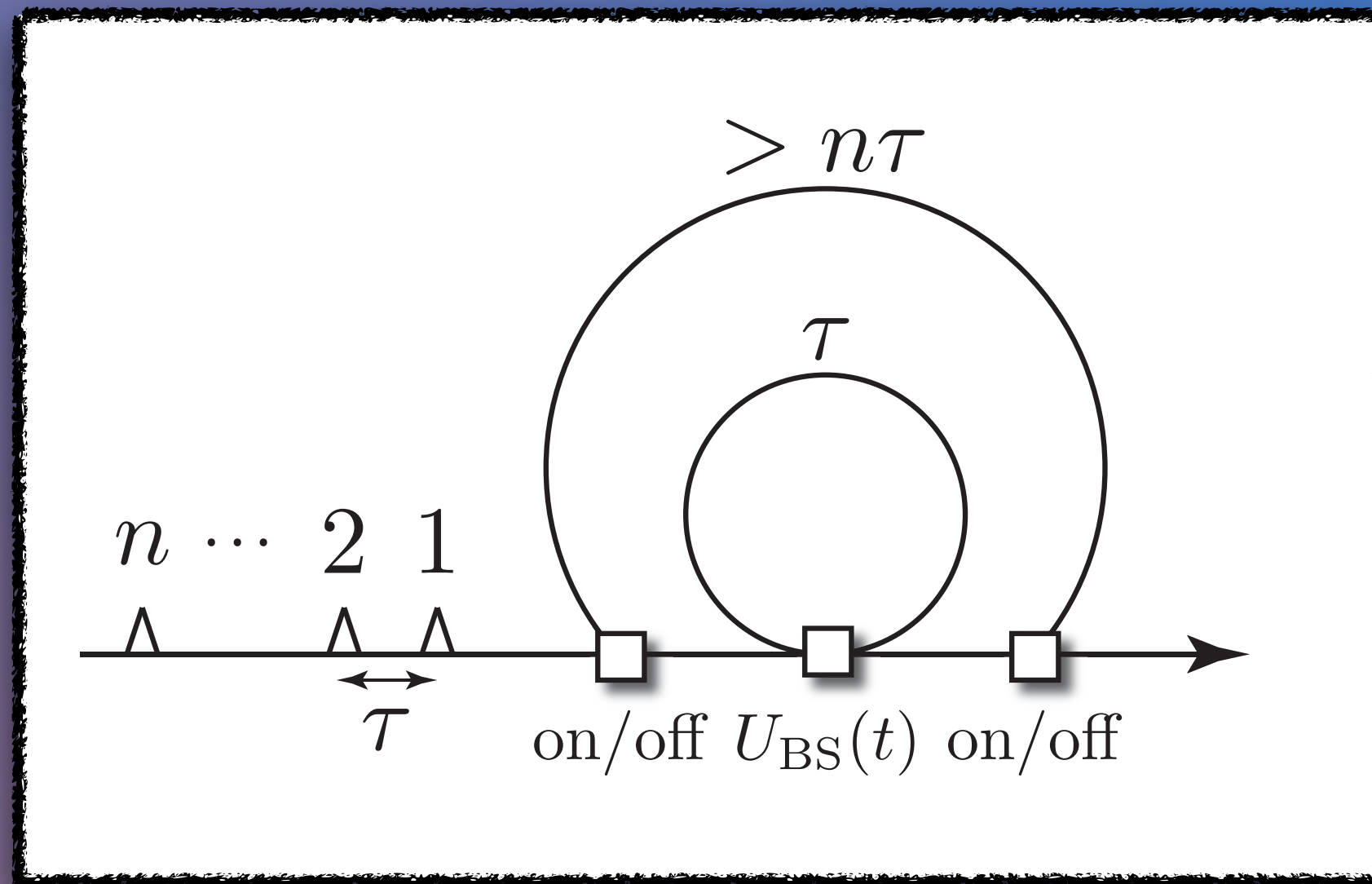
SPACE



TIME

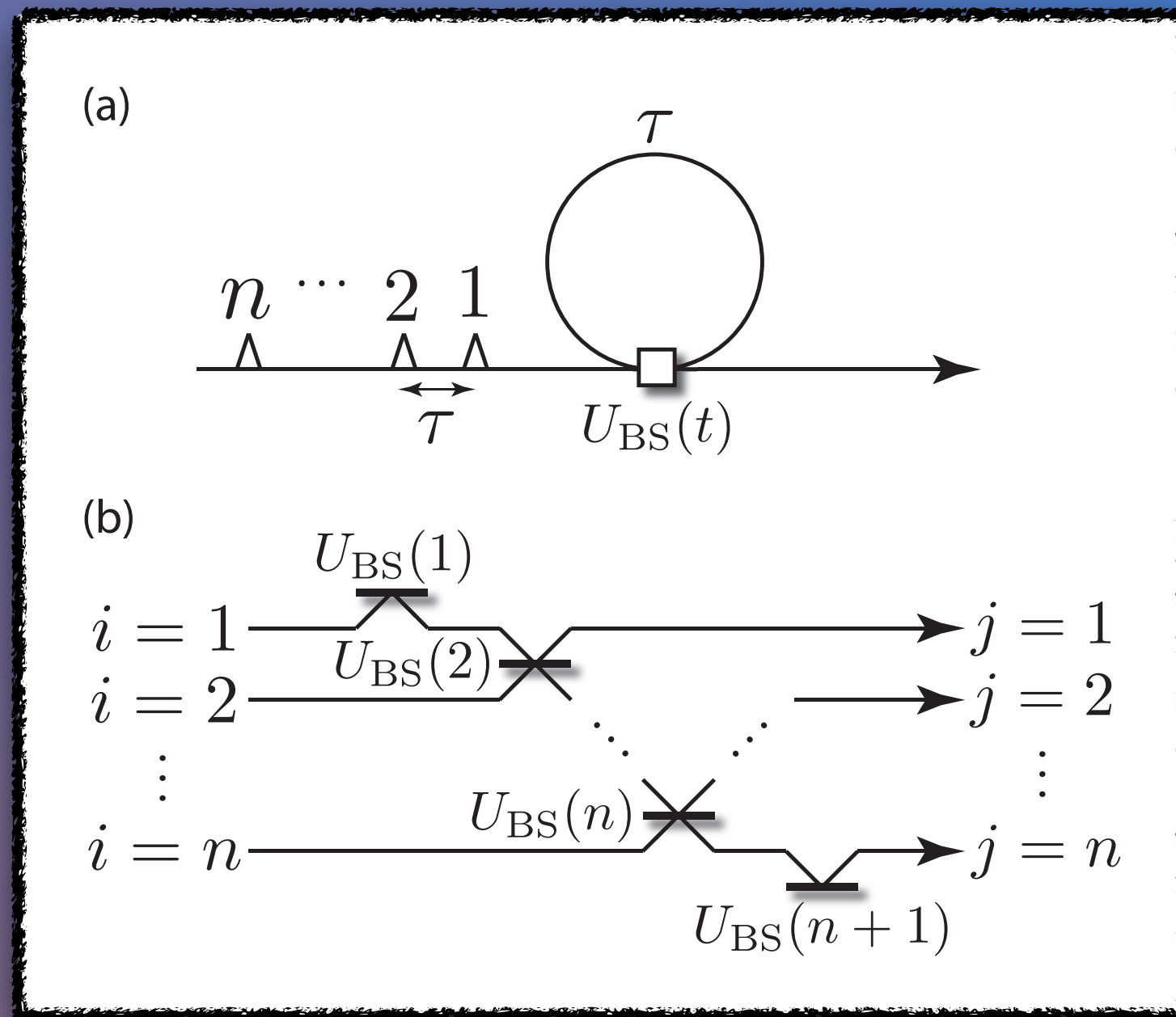


FULL FIBERLOOP ARCHITECTURE

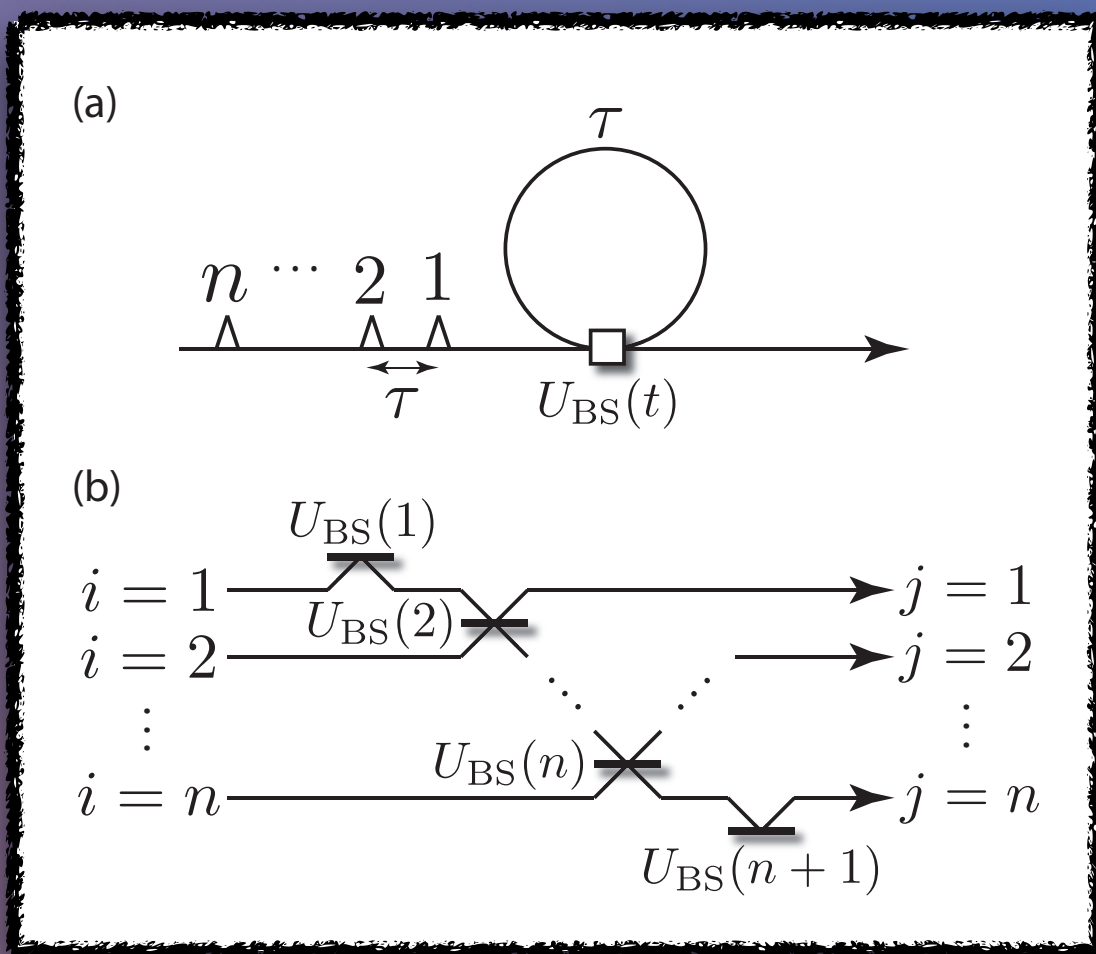


HOW DOES IT WORK?

SINGLE LOOP



INNER LOOP TRANSFORMATION

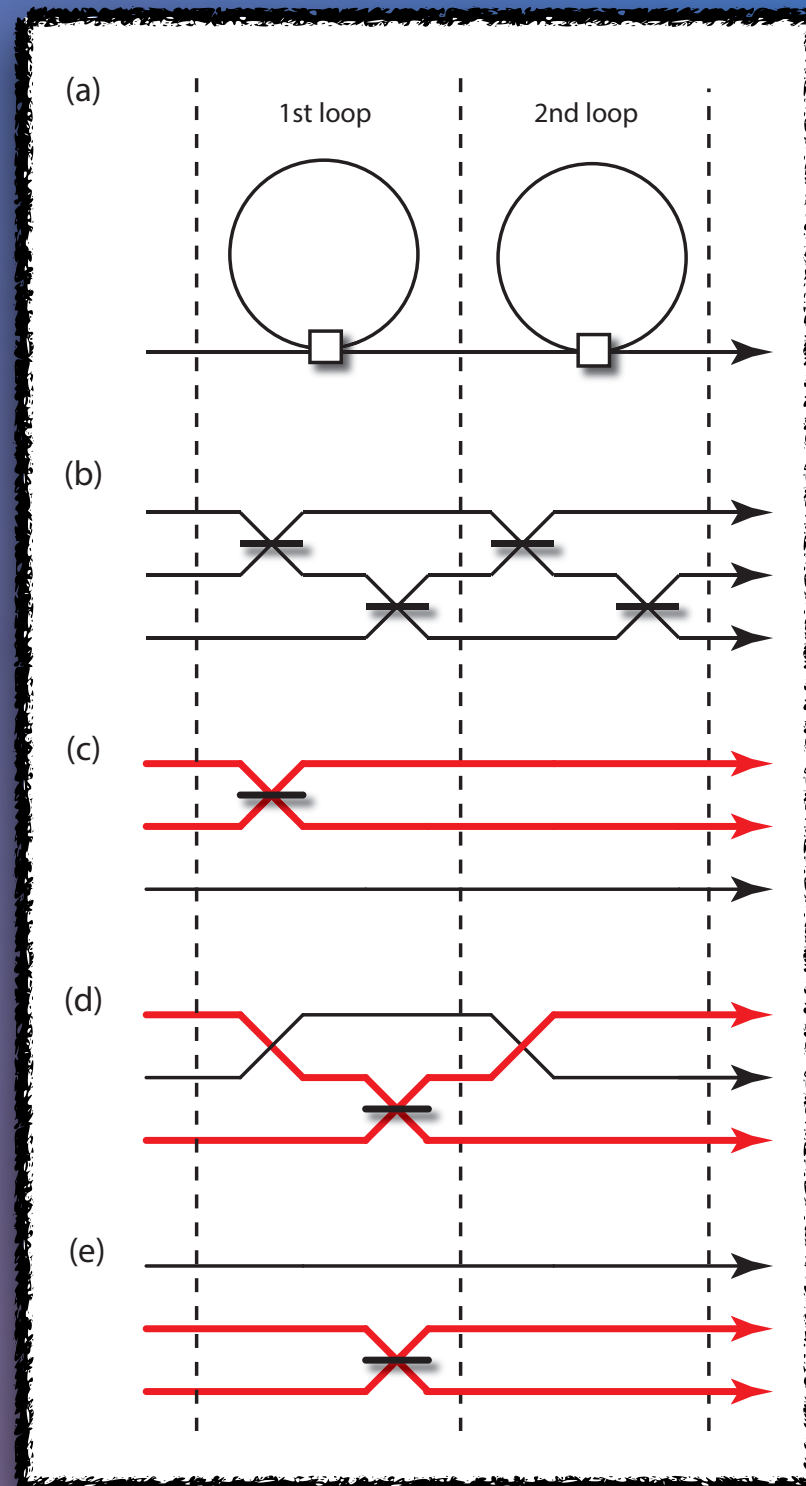


$$\hat{U}_{\text{BS}}(t) = \begin{pmatrix} u_{1,1}(t) & u_{1,2}(t) \\ u_{2,1}(t) & u_{2,2}(t) \end{pmatrix}$$

$$V_{i,j} = \begin{cases} 0 & i > j + 1 \\ u_{1,1}(i) & i = j + 1 \\ u_{1,2}(i)u_{2,1}(j + 1) \prod_{k=i+1}^j u_{2,2}(k) & i < j + 1 \end{cases}$$

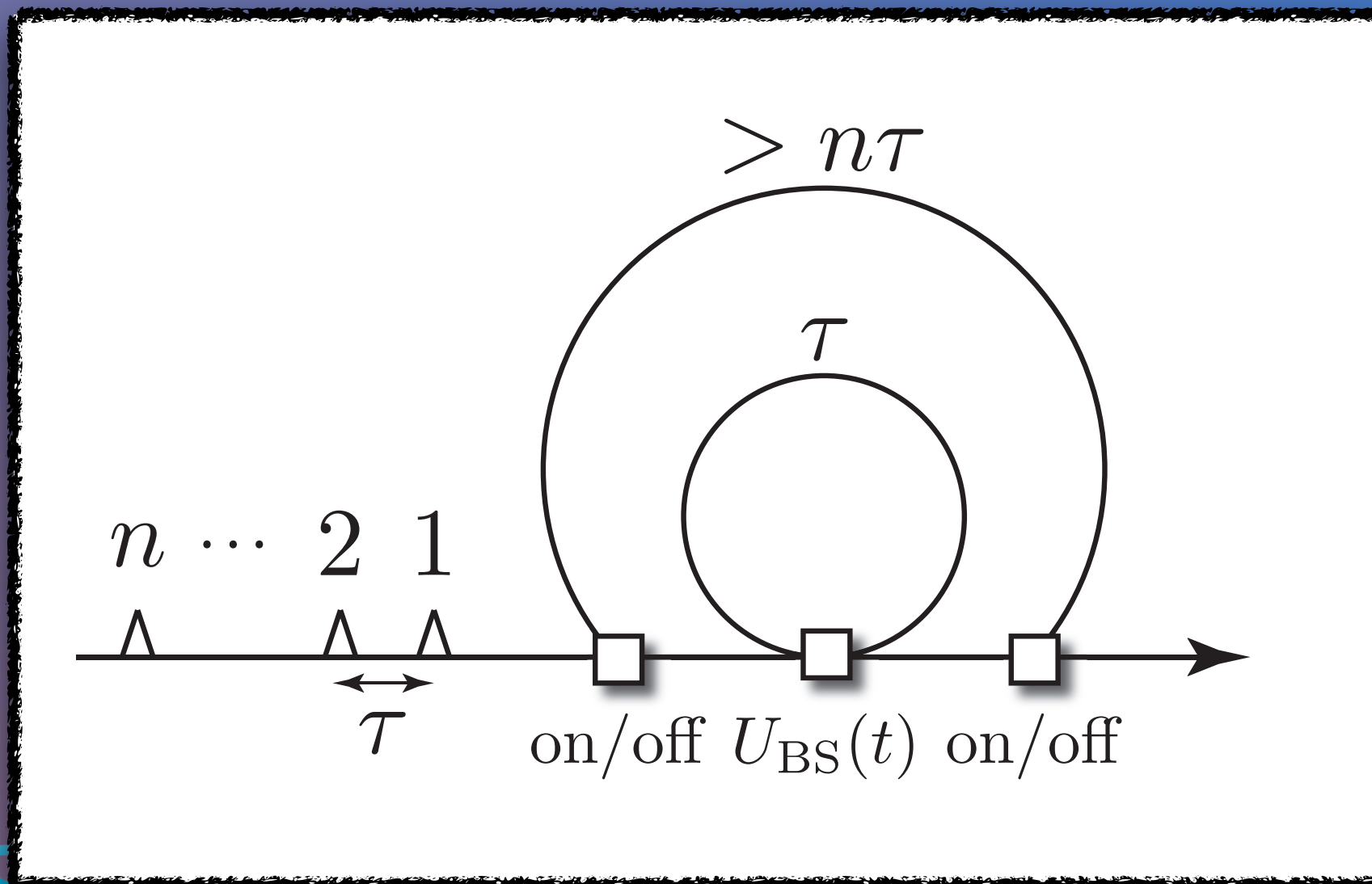


EVOLUTION

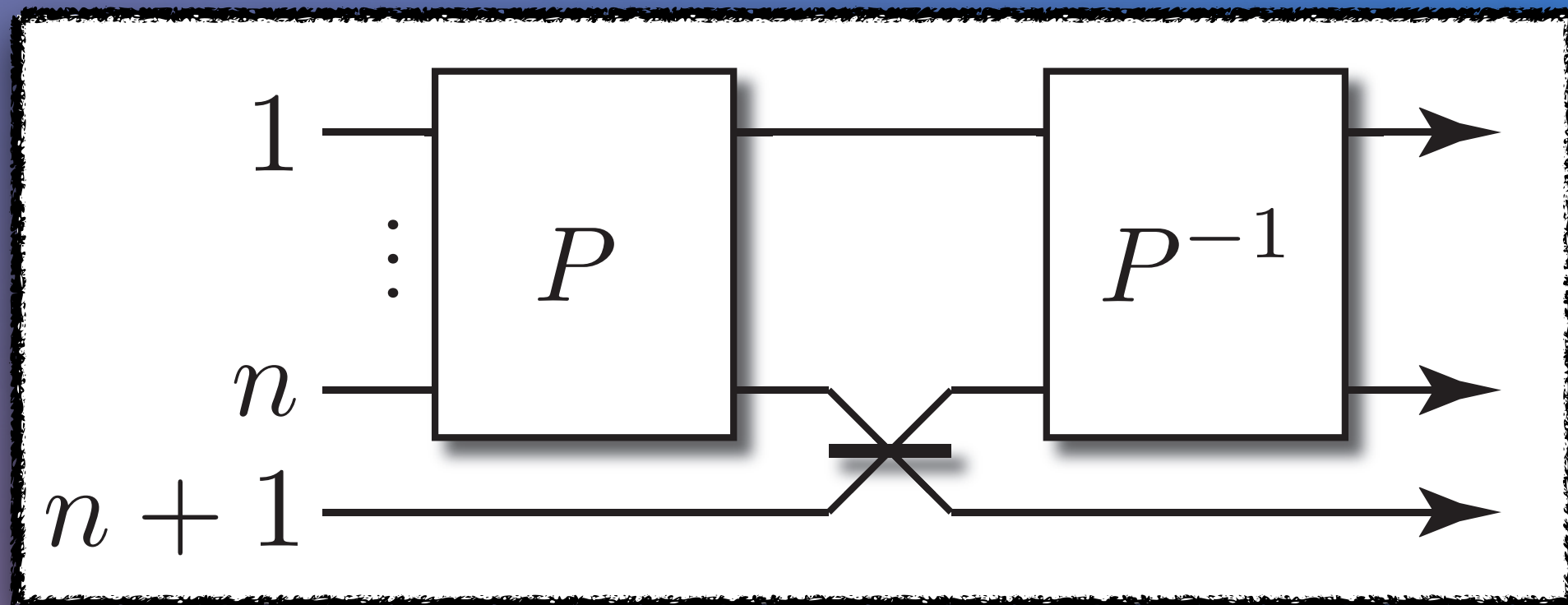


FULL ARCHITECTURE

$$\hat{U} = \prod_{l=1}^L \hat{V}(l) \quad V_{i,j} = \begin{cases} 0 & i > j + 1 \\ u_{1,1}(i) & i = j + 1 \\ u_{1,2}(i)u_{2,1}(j + 1) \prod_{k=i+1}^j u_{2,2}(k) & i < j + 1 \end{cases}$$

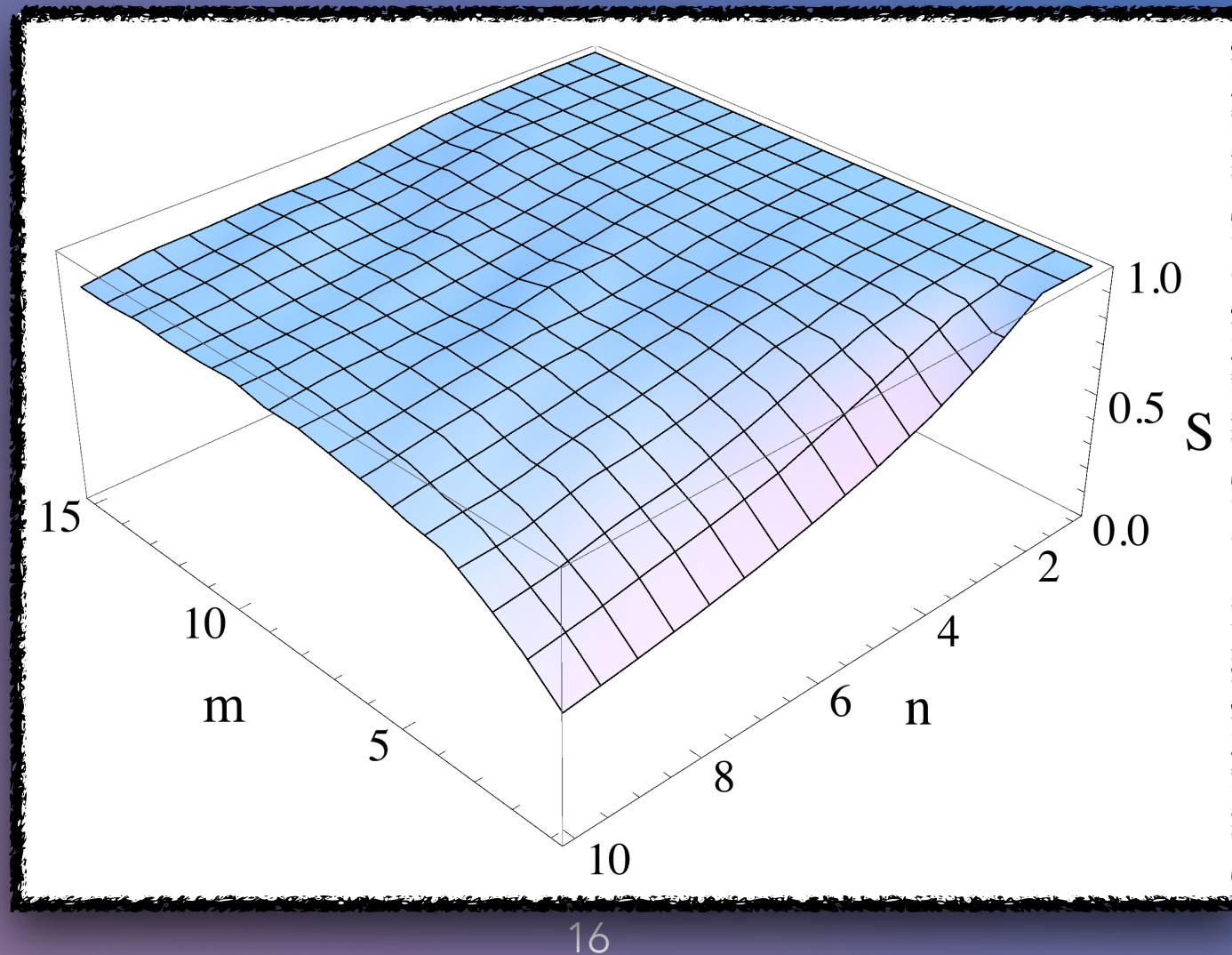


INDUCTIVE ARGUMENT



SIMILARITY

$$S = \max_{\hat{U}_{\text{BS}}(t) \forall t} \left[\frac{\left(\sum_{i,j} \sqrt{|U_{i,j}|^2 \cdot |U'_{i,j}|^2} \right)^2}{\left(\sum_{i,j} |U_{i,j}|^2 \right) \cdot \left(\sum_{i,j} |U'_{i,j}|^2 \right)} \right] = \max_{\hat{U}_{\text{BS}}(t) \forall t} \left[\frac{1}{n^3} \left(\sum_{i,j} |U_{i,j}| \right)^2 \right]$$



ERROR ANALYSIS: LOSS AND MODE MISMATCH



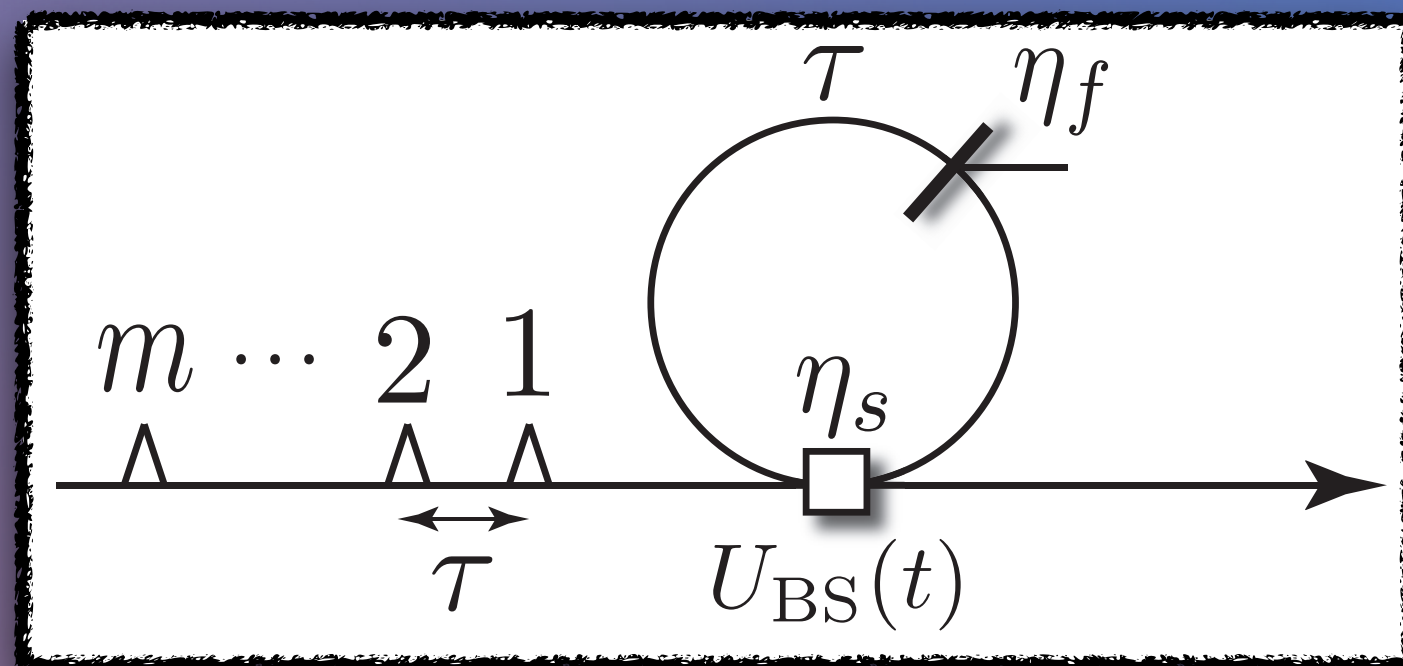
LOSS IN THE INNER LOOP

$$V_{i,j} = \begin{cases} 0 & i > j + 1 \\ u_{1,1}(i) & i = j + 1 \\ u_{1,2}(i)u_{2,1}(j + 1) \prod_{k=i+1}^j u_{2,2}(k) & i < j + 1 \end{cases}$$

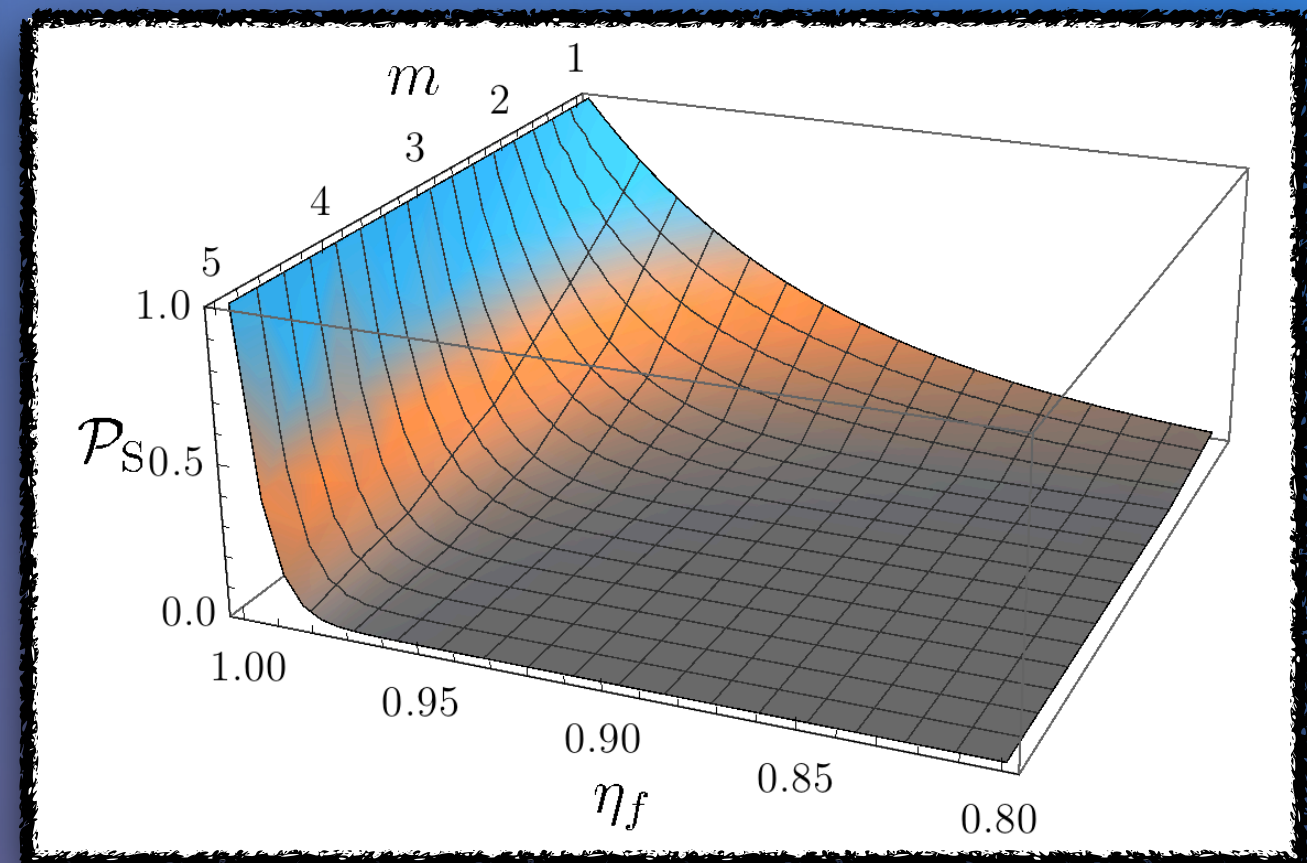
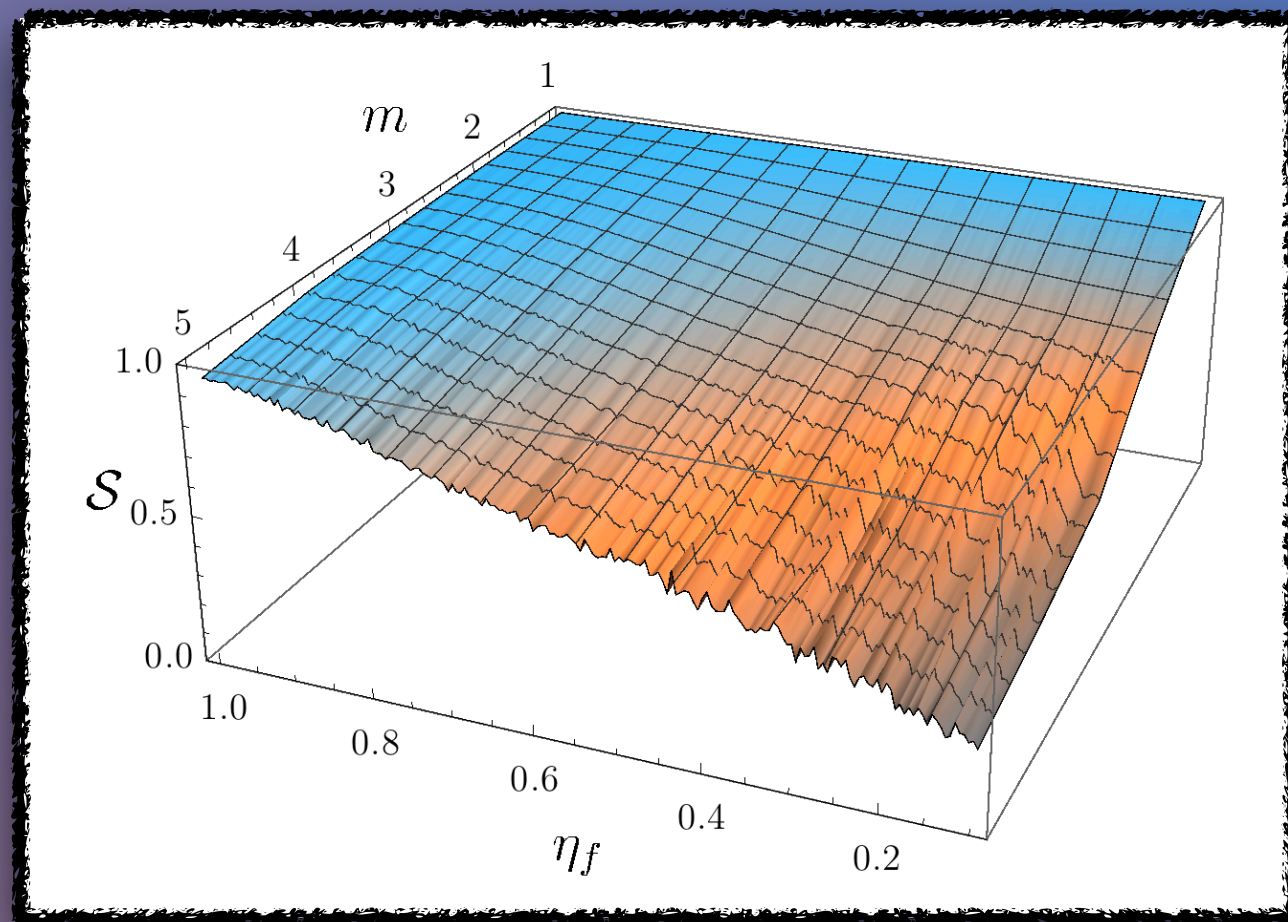
$$V'_{i,j} = \eta_s \begin{cases} 0 & i > j + 1 \\ u_{1,1}(i) & i = j + 1 \\ \eta^{j-i+1} u_{1,2}(i) u_{2,1}(j + 1) \cdot & \\ \prod_{k=i+1}^j u_{2,2}(k) & i < j + 1 \end{cases}$$

$$\eta = \eta_f \eta_s$$

$\eta f \eta s^2$	$\eta f^2 \eta s^3$	$\eta f^3 \eta s^4$	$\eta f^4 \eta s^5$	$\eta f^5 \eta s^6$
ηs	$\eta f \eta s^2$	$\eta f^2 \eta s^3$	$\eta f^3 \eta s^4$	$\eta f^4 \eta s^5$
0	ηs	$\eta f \eta s^2$	$\eta f^2 \eta s^3$	$\eta f^3 \eta s^4$
0	0	ηs	$\eta f \eta s^2$	$\eta f^2 \eta s^3$
0	0	0	ηs	$\eta f \eta s^2$

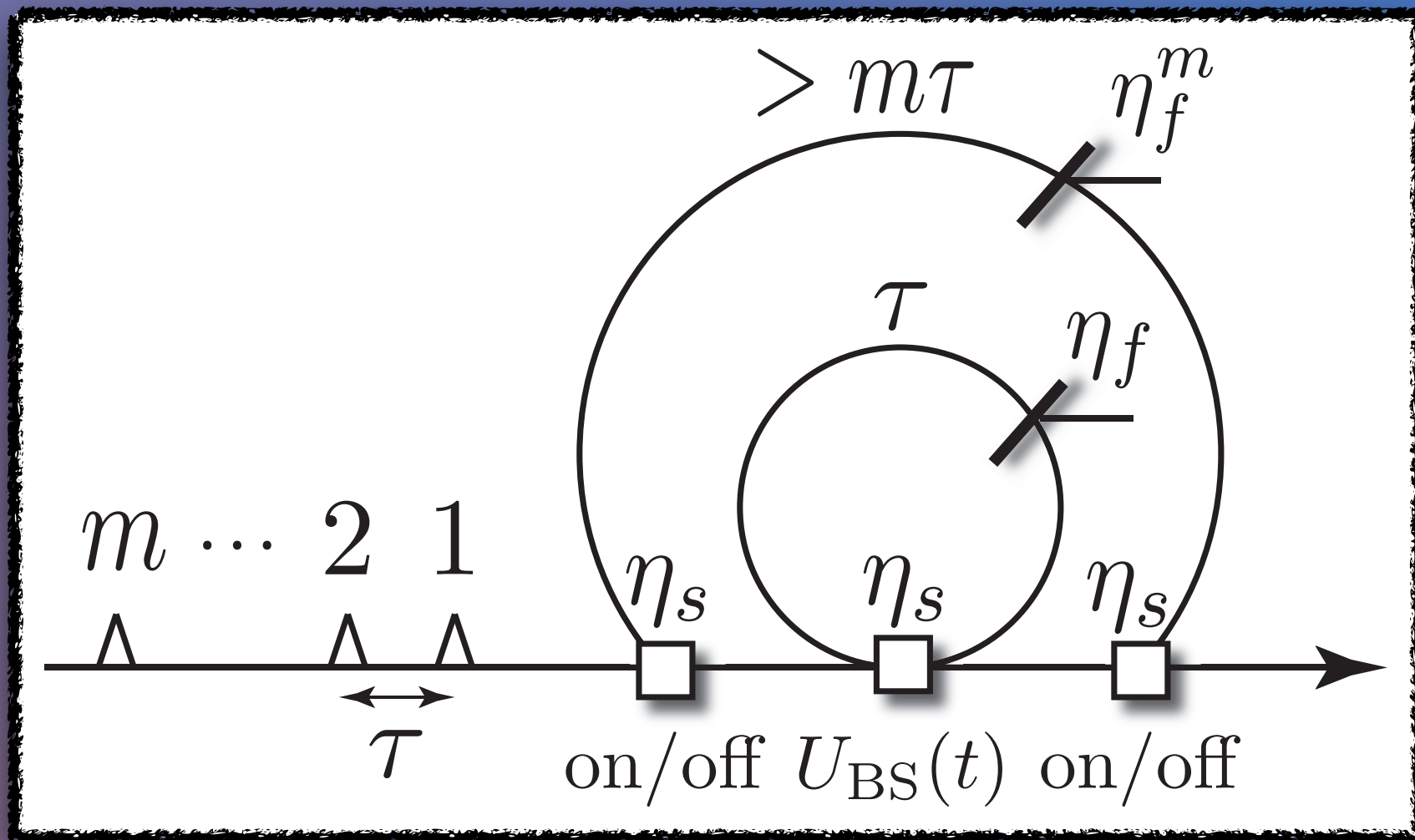


LOSS IN THE INNER LOOP

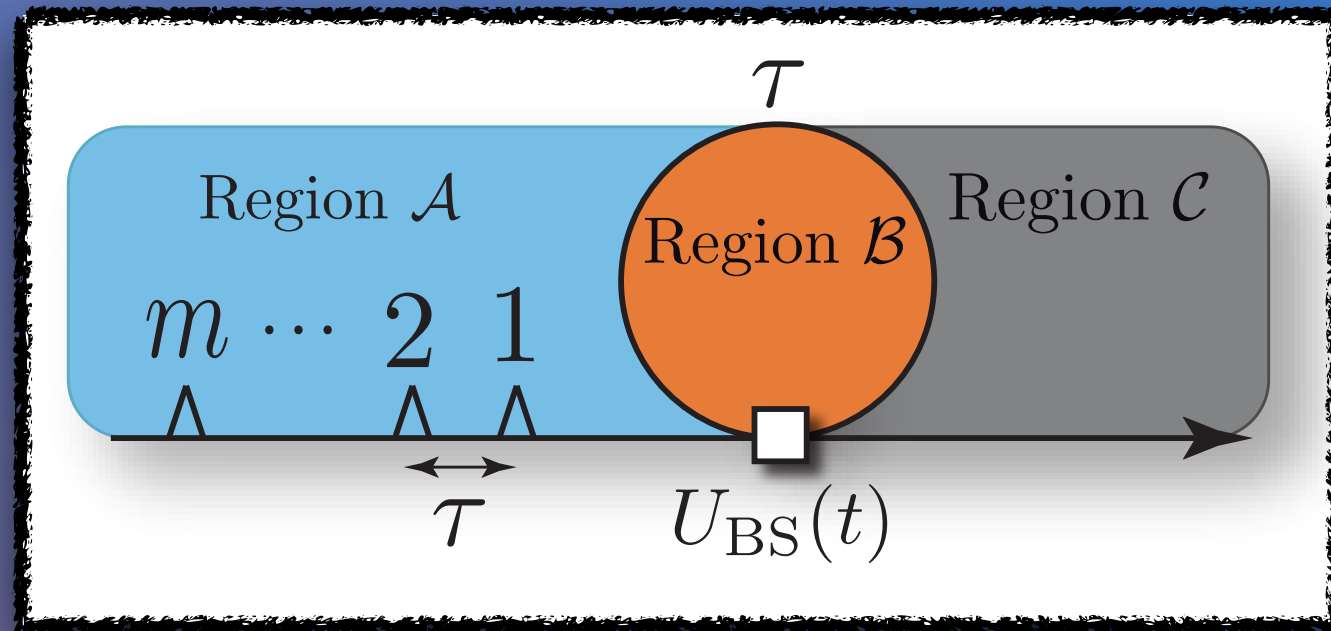


LOSS IN THE FULL ARCHITECTURE

$$\hat{U} = \prod_{l=1}^L \hat{V}(l) \qquad \hat{U}' = \eta_f^{m(L-1)} \eta_s^{2(L-1)} \prod_{l=1}^L \hat{V}'(l)$$



MODE MISMATCH



$$\hat{\mathcal{A}}^\dagger(t, \Delta) = \int_{-\infty}^{\infty} \psi(x - t - \Delta) \hat{a}^\dagger(x) dx$$

$$|\Psi_{\text{in}}\rangle = \bigotimes_{i=1}^m \frac{1}{\sqrt{k_i!}} \hat{\mathcal{A}}^\dagger(t_i, \Delta_i)^{k_i} |0\rangle_i$$

$$\hat{U}_{BS}(t) \hat{\mathcal{A}}^\dagger(t, \Delta) \hat{U}_{BS}^\dagger(t) \rightarrow u_{1,2}(t) \hat{\mathcal{B}}^\dagger(t+1, \Delta) + u_{1,1}(t) \hat{\mathcal{C}}^\dagger(t, \Delta)$$

$$\hat{U}_{BS}(t) \hat{\mathcal{B}}^\dagger(t, \Delta) \hat{U}_{BS}^\dagger(t) \rightarrow u_{2,2}(t) \hat{\mathcal{B}}^\dagger(t+1, \Delta) + u_{2,1}(t) \hat{\mathcal{C}}^\dagger(t, \Delta)$$

METRIC: FIDELITY

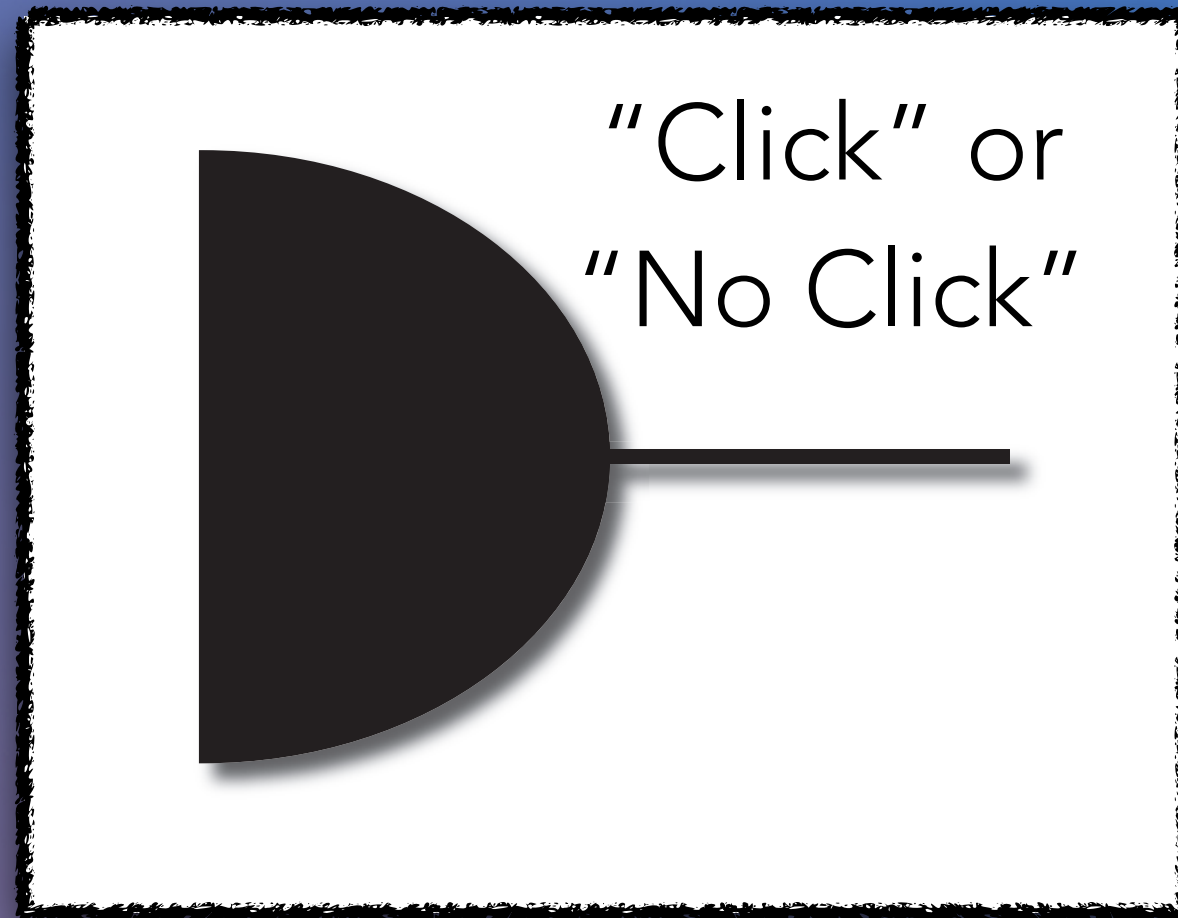
$$\mathcal{F} = |\langle \Psi_i | \Psi_a \rangle|^2$$

...

$$\mathcal{F} = \left| \sum_{S', S} \sum_{T'(S'), T(S)} \gamma_{S', T'} \cdot \gamma_{S, T} \cdot m! \cdot \exp\left(-\frac{m \Delta_{T(S_i)}^2}{4c^2}\right) \right|^2$$

DETECTION

BUCKET DETECTORS WITH POST-SELECTION UPON
DETECTING ALL PHOTONS IS SUFFICIENT



WHY?

OF MODES $\gg$ # OF INPUT PHOTONS





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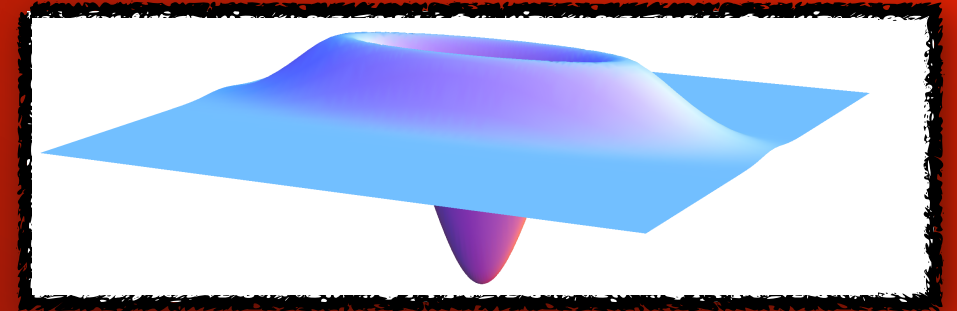
BOSON-SAMPLING WITH NON-FOCK STATES

BY: KEITH R. MOTES, JONATHAN P. OLSON,
KAUSHIK P. SESHADREESAN, PAUL KNOTT,
JOSEPH FITZSIMONS, WILLIAM MUNRO,
JONATHAN P. DOWLING & PETER P. ROHDE

SAMPLING OTHER QUANTUM STATES OF LIGHT

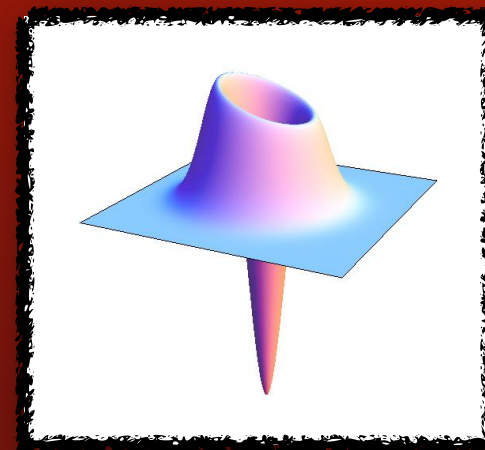
PHOTON-ADDED OR SUBTRACTED
SQUEEZED VACUUM STATES:

ARXIV:1406.7821 (2014)



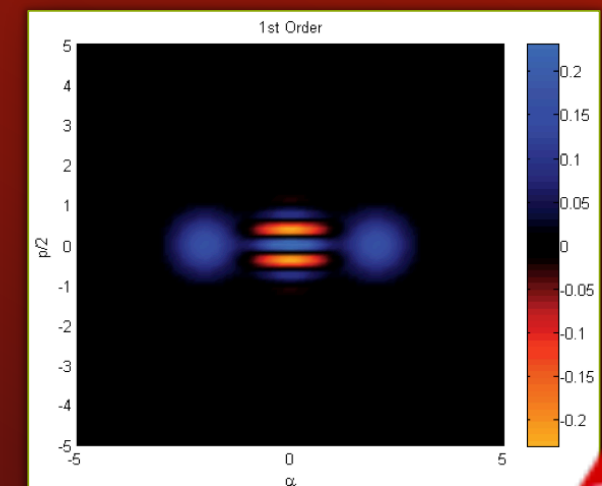
SINGLE PHOTON-ADDED
COHERENT STATES:

ARXIV:1402.0531 (2014)



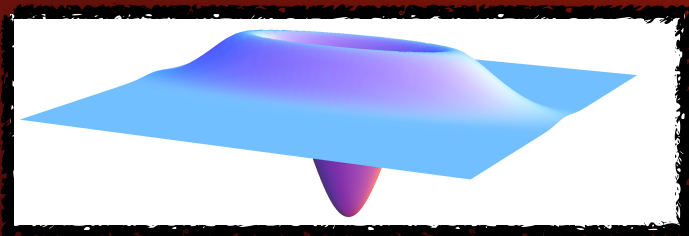
GENERALIZED
CAT STATES:

ARXIV:1310.0297 (2013)



PHOTON-ADDED OR SUBTRACTED SQUEEZED VACUUM STATES

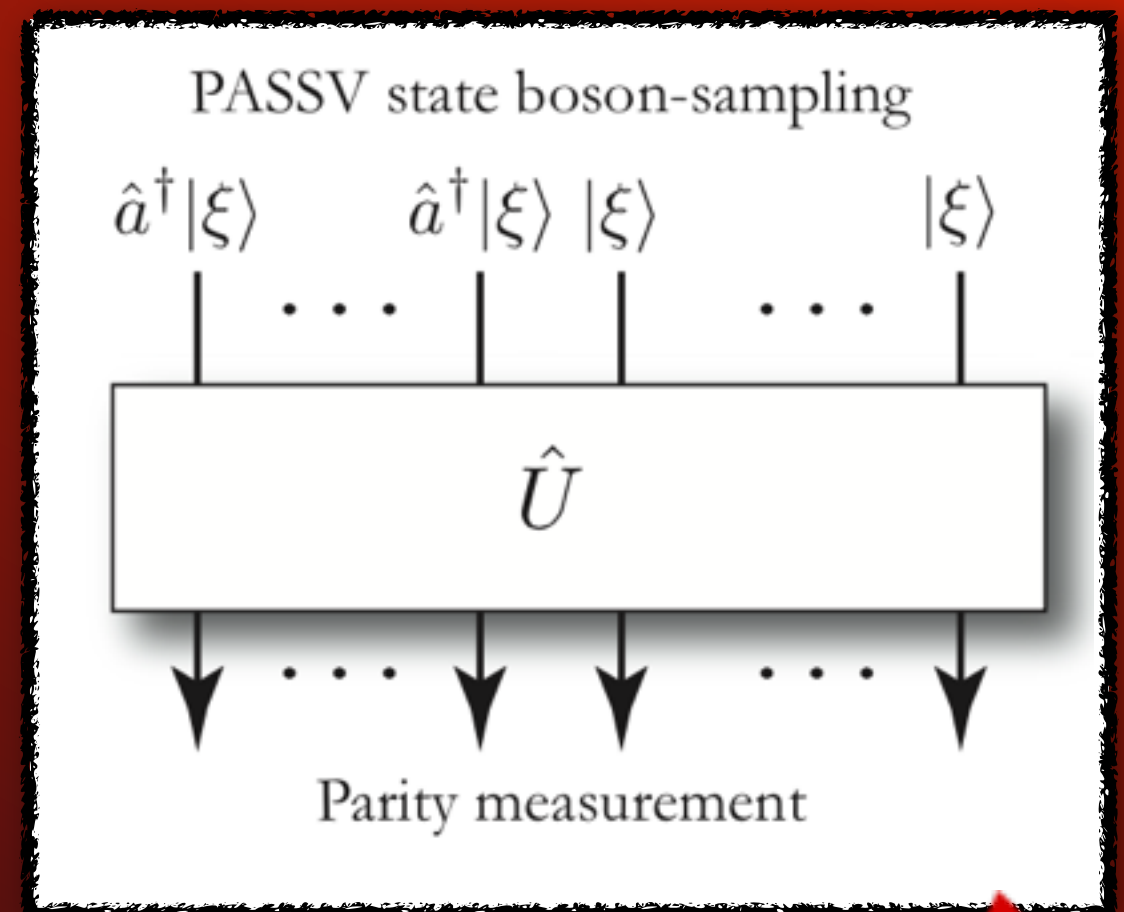
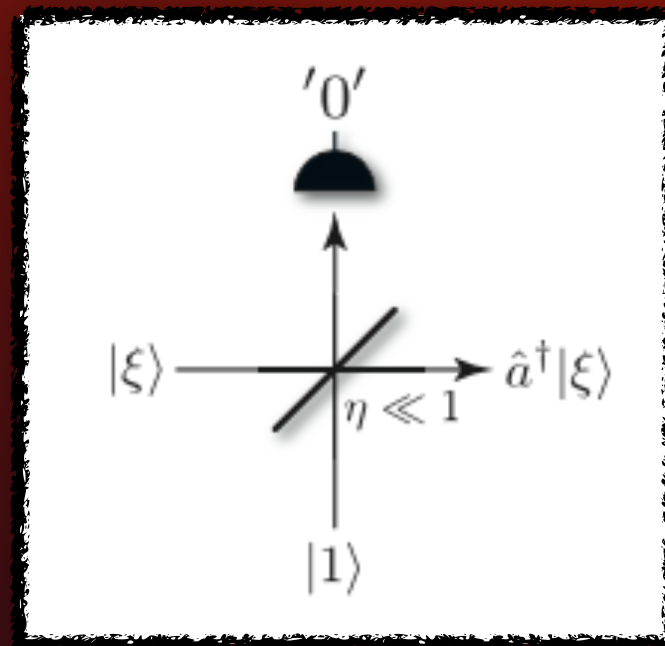
DEFINITION: $|PASSV\rangle = \hat{a}^\dagger \hat{S}(\xi)|0\rangle = \hat{a}^\dagger |\xi\rangle$



$$\hat{S}(\xi) = \exp\left(\frac{1}{2}(\xi^* \hat{a}^2 - \xi \hat{a}^{\dagger 2})\right)$$

MODEL:

GENERATION:



PHOTON-ADDED OR SUBTRACTED SQUEEZED VACUUM STATES

COMPUTATIONALLY HARD FOR ALL
SQUEEZING PARAMETERS

$$|\psi\rangle_{\text{in}} = \hat{a}_1^\dagger \dots \hat{a}_n^\dagger |\xi_1, \dots, \xi_m\rangle$$

$$|\psi\rangle_{\text{out}} = \hat{U} |\psi\rangle_{\text{in}}$$

$$= \hat{U} \left(\hat{a}_1^\dagger \dots \hat{a}_n^\dagger \right) \hat{U}^\dagger \hat{U} |\xi_1, \dots, \xi_m\rangle$$

$$= \hat{U} \left(\hat{a}_1^\dagger \dots \hat{a}_n^\dagger \right) \hat{U}^\dagger |\xi_1, \dots, \xi_m\rangle$$

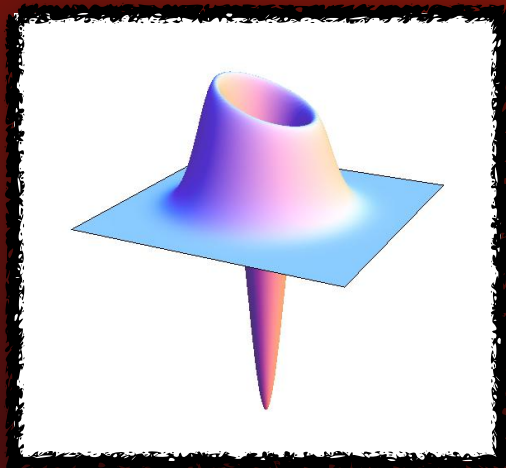
$$= \widehat{AA} |\xi_1, \dots, \xi_m\rangle$$

$\hat{U}$ is composed of real beam-
splitters and is orthogonal so
that the squeezed and anti-
squeezed quadratures don't
mix.



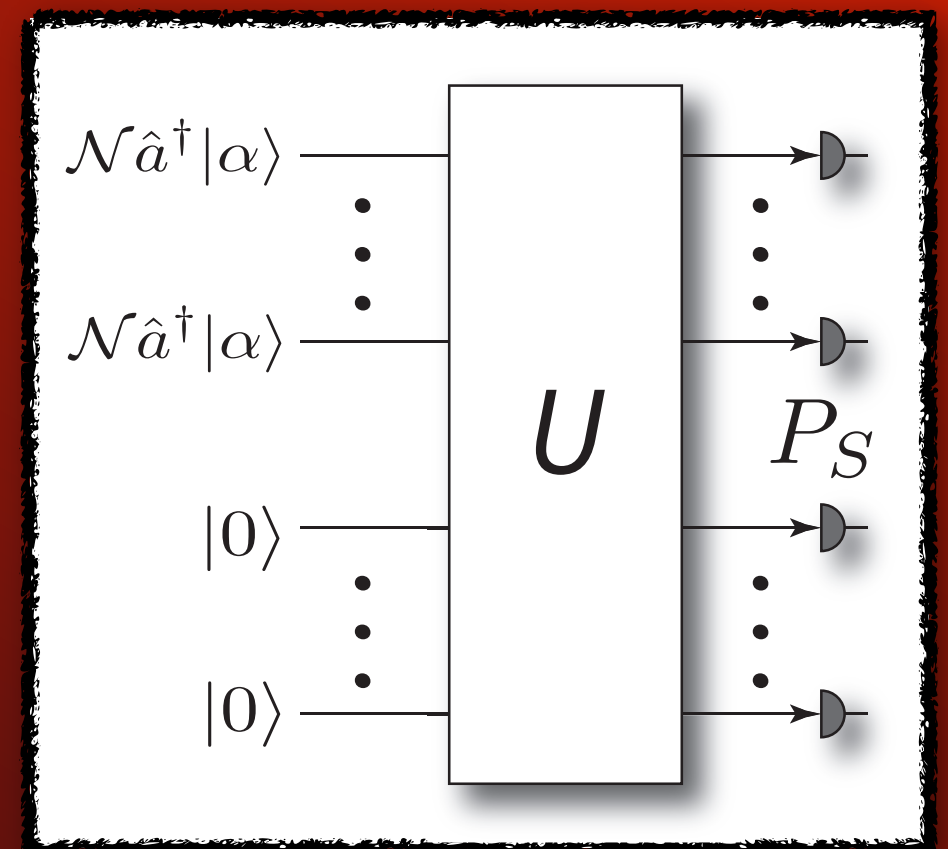
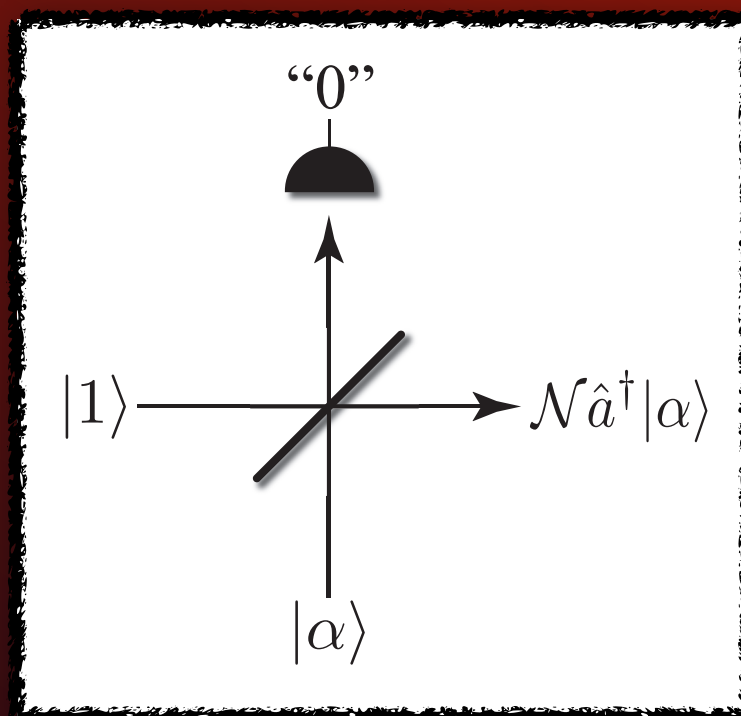
SINGLE PHOTON-ADDED COHERENT STATES

DEFINITION: $|SPACS\rangle = \mathcal{N}\hat{a}^\dagger|\alpha\rangle$



MODEL:

GENERATION:



SINGLE PHOTON-ADDED COHERENT STATES

WE USE THE RESULT FOR
APPROXIMATE BOSON-SAMPLING

$$|\psi_{\text{out}}\rangle = \mathcal{N} \prod_{i=1}^n \left(\underbrace{\hat{U} \hat{a}_i^\dagger \hat{U}^\dagger}_{\widehat{AA}} + \alpha^{(i)*} \right) |0_1, \dots, 0_m\rangle$$

$$P(n) = \frac{1}{(1 + |\alpha|^2)^n} \geq \frac{1}{\text{poly}(n)}$$



SINGLE PHOTON-ADDED COHERENT STATES

THREE COMPUTATIONAL LIMITS

HARD:

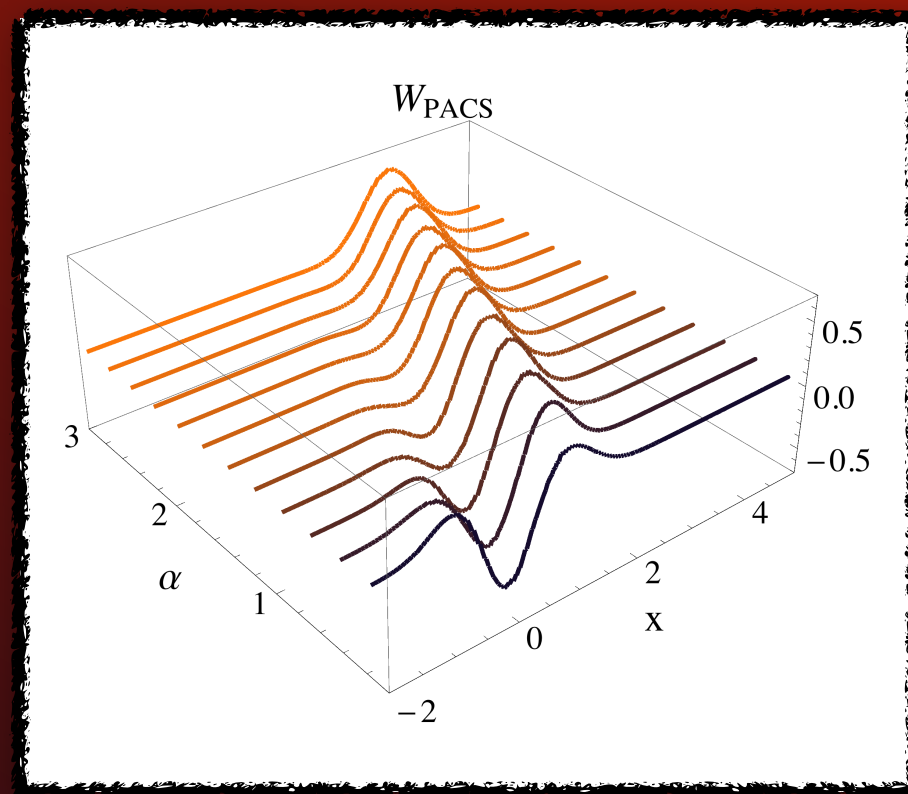
$$|\alpha|^2 \leq \frac{1}{n}$$

INTERMEDIATE:

$$\frac{1}{n} \leq |\alpha|^2 \leq n^2$$

EASY:

$$|\alpha|^2 \geq n^2$$



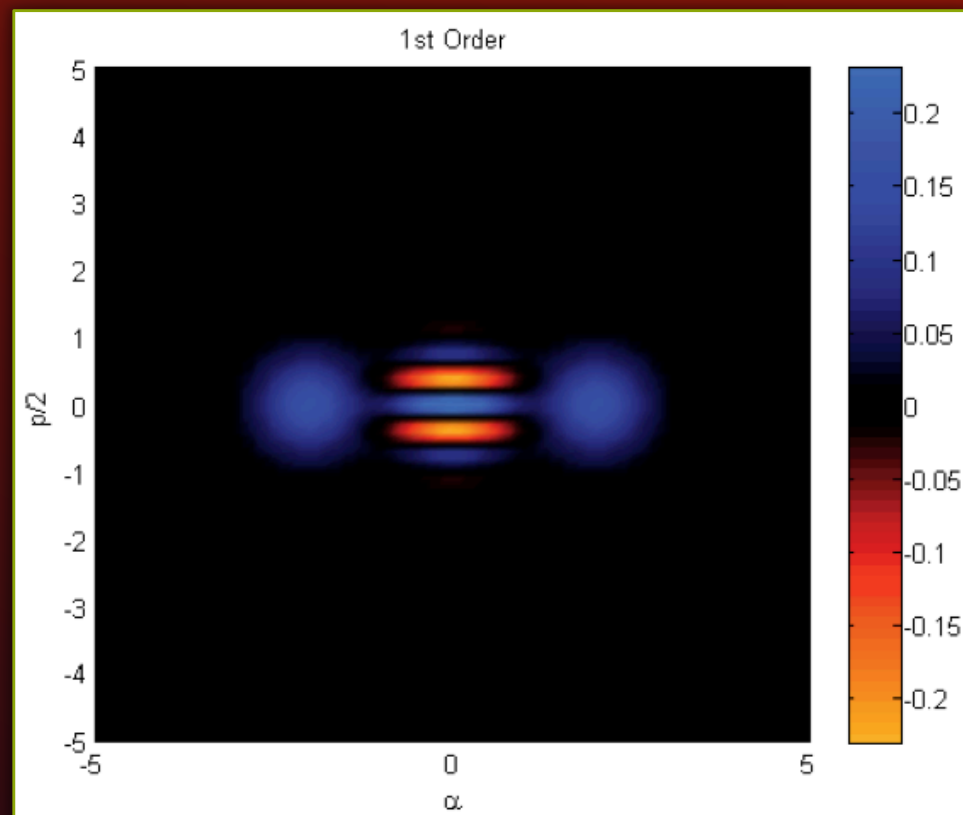
ARXIV:1402.0531 (2014)



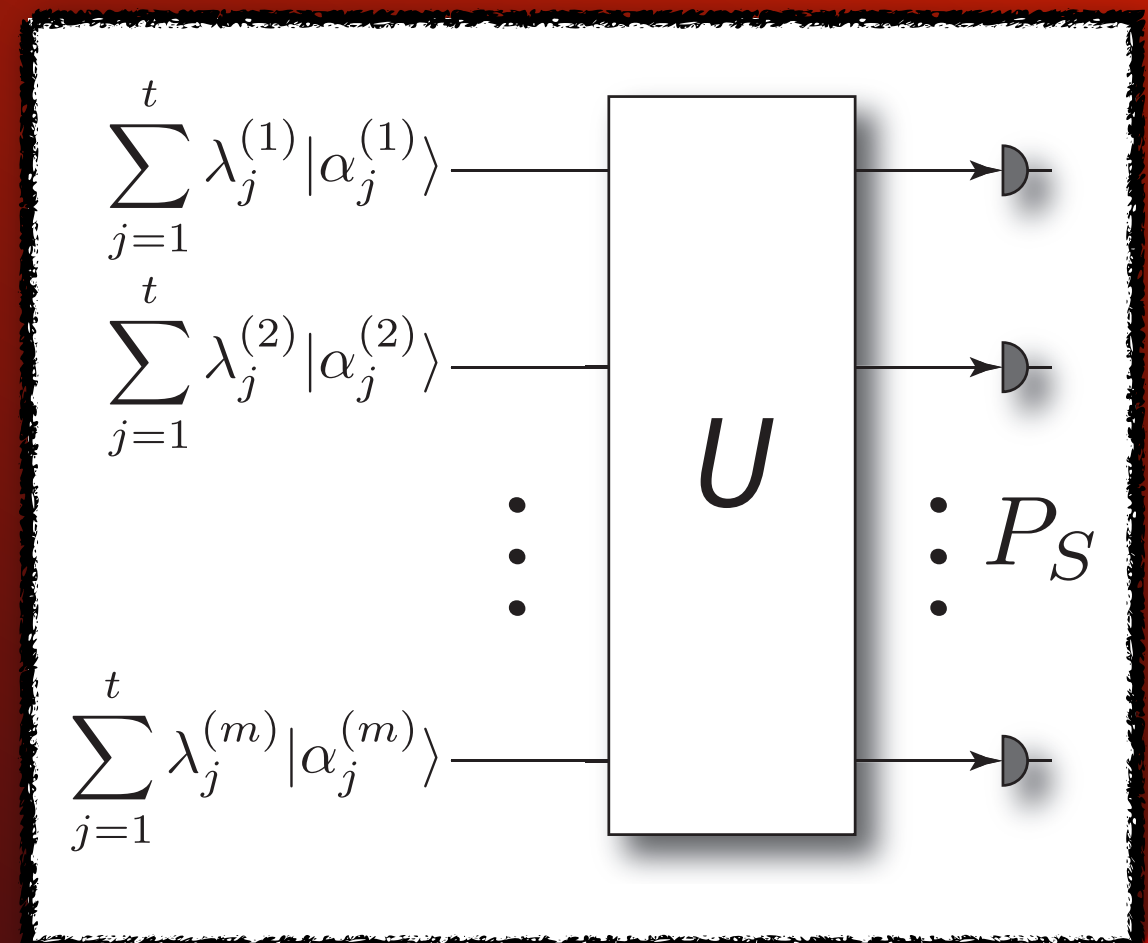
GENERALIZED CAT STATES

DEFINITION:

$$|\text{cat}\rangle = \sum_{j=1}^t \lambda_j |\alpha_j\rangle$$



MODEL:



GENERALIZED CAT STATES

COMPARISON TO PERMANENT:

$$\gamma_S = \sum_{\vec{t}=1}^t \prod_{j=1}^m A_{j,\vec{t}}^{(S)} \quad \text{Perm}(A) = \sum_{\sigma \in S_n} \prod_{i=1}^n A_{i,\sigma(i)}$$

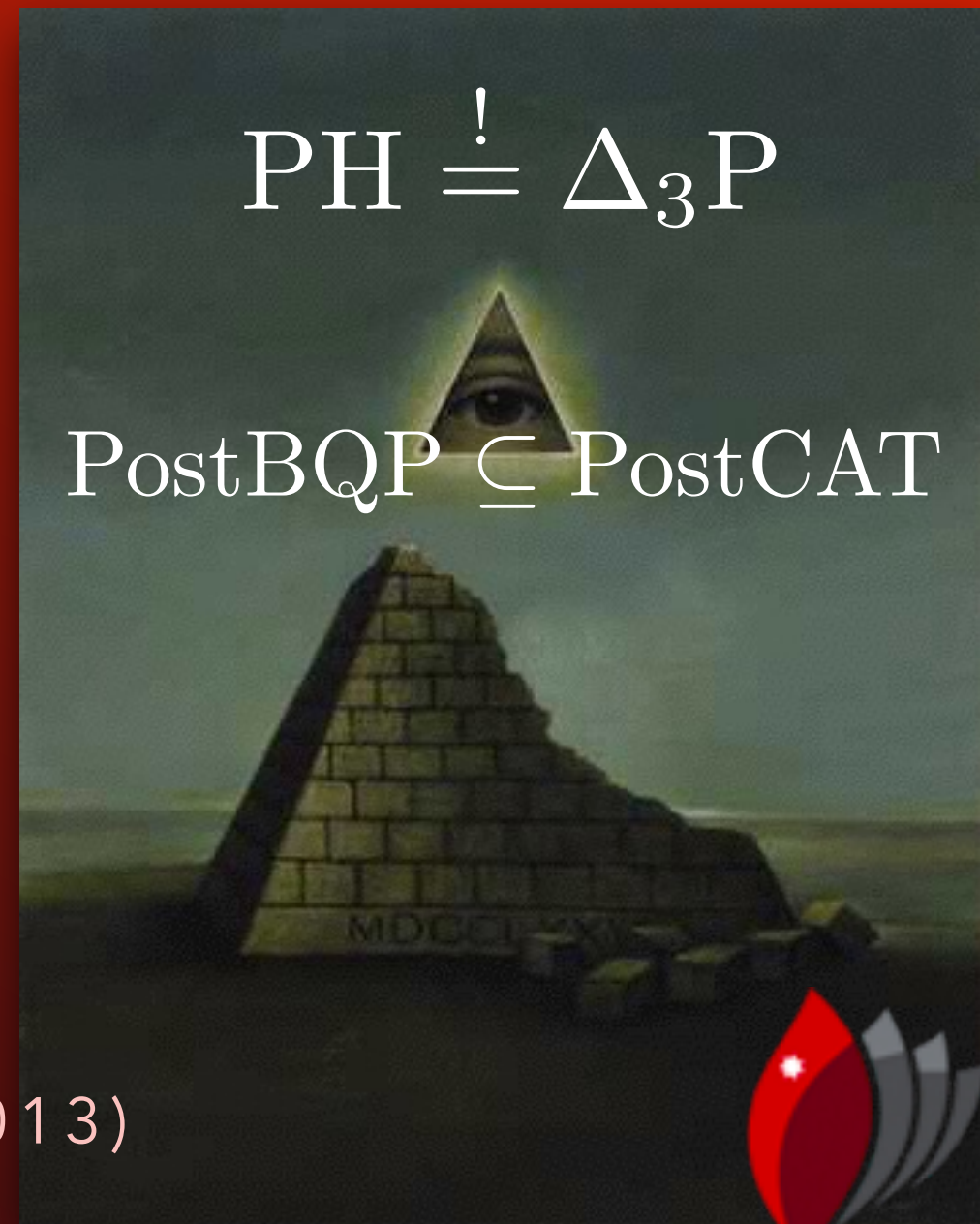
$$A_{j,\vec{t}}^{(S)} = \lambda_{t_j}^{(j)} f_{S_j} \left(\sum_{k=1}^m U_{j,k} \alpha_{t_j}^{(k)} \right)$$

$$f_n(\alpha) = e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}}$$

COMPUTATIONAL
COMPLEXITY ARGUMENT:

$$\text{PH} \stackrel{!}{=} \Delta_3\text{P}$$

$$\text{PostBQP} \subseteq \text{PostCAT}$$



MANY QUANTUM STATES OF LIGHT ARE ALSO
COMPUTATIONALLY HARD TO SAMPLE





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**LINEAR OPTICAL QUANTUM
METROLOGY WITH SINGLE PHOTONS:
EXPLOITING SPONTANEOUSLY
GENERATED ENTANGLEMENT TO BEAT
THE SHOTNOISE LIMIT**

KEITH R. MOTES, JONATHAN P. OLSON, EVAN J.
RABEAUX, JONATHAN P. DOWLING, S. JAY OLSON
& PETER P. ROHDE

PRL 114, 170802

WHY BOSON-SAMPLING?

nature
photonics

ARTICLES

PUBLISHED ONLINE: 24 AUGUST 2015 | DOI: 10.1038/NPHOTON.2015.153

Boson sampling for molecular vibronic spectra

Joonsuk Huh*, Gian Giacomo Guerreschi, Borja Peropadre, Jarrod R. McClean
and Alán Aspuru-Guzik*

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Linear Optical Quantum Metrology with Single Photons: Exploiting Spontaneously Generated Entanglement to Beat the Shot-Noise Limit

Keith R. Motes, Jonathan P. Olson, Evan J. Rabeaux, Jonathan P. Dowling, S. Jay Olson, and
Peter P. Rohde

Phys. Rev. Lett. **114**, 170802 – Published 30 April 2015



yes..for this we must visit the
dark lord Sauron



LOUISIANA STATE UNIVERSITY



THE LABORATORY



Tech Edge
by Jenny Linscott & Paige Brown



MEET SAURON

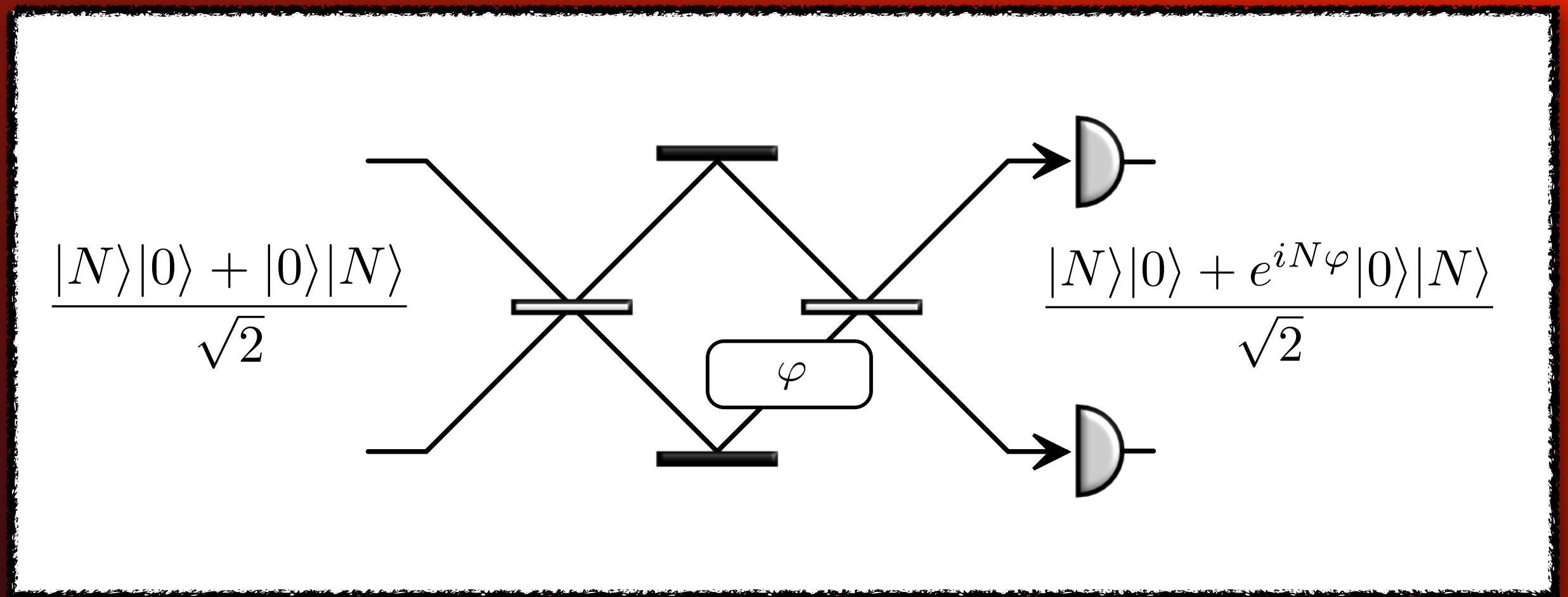


Okay time to go
brainstorm...be right
back

Grants
nt



NOON STATE METROLOGY



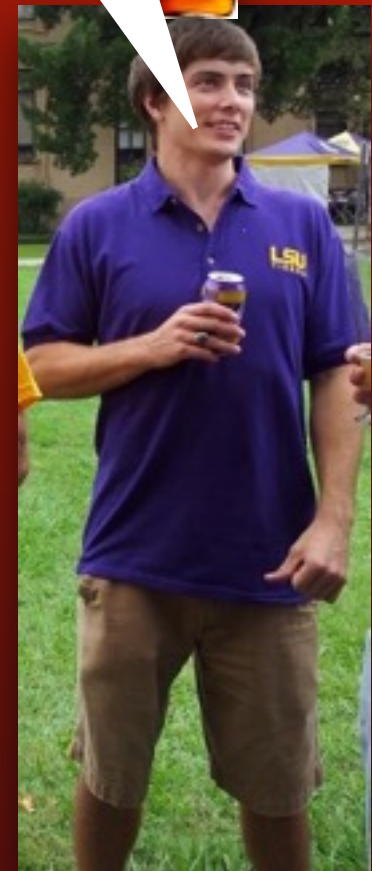
THIS IS OPTIMAL FOR QUANTUM METROLOGY

$$\varphi_{\text{NOON}} = \frac{1}{N}$$

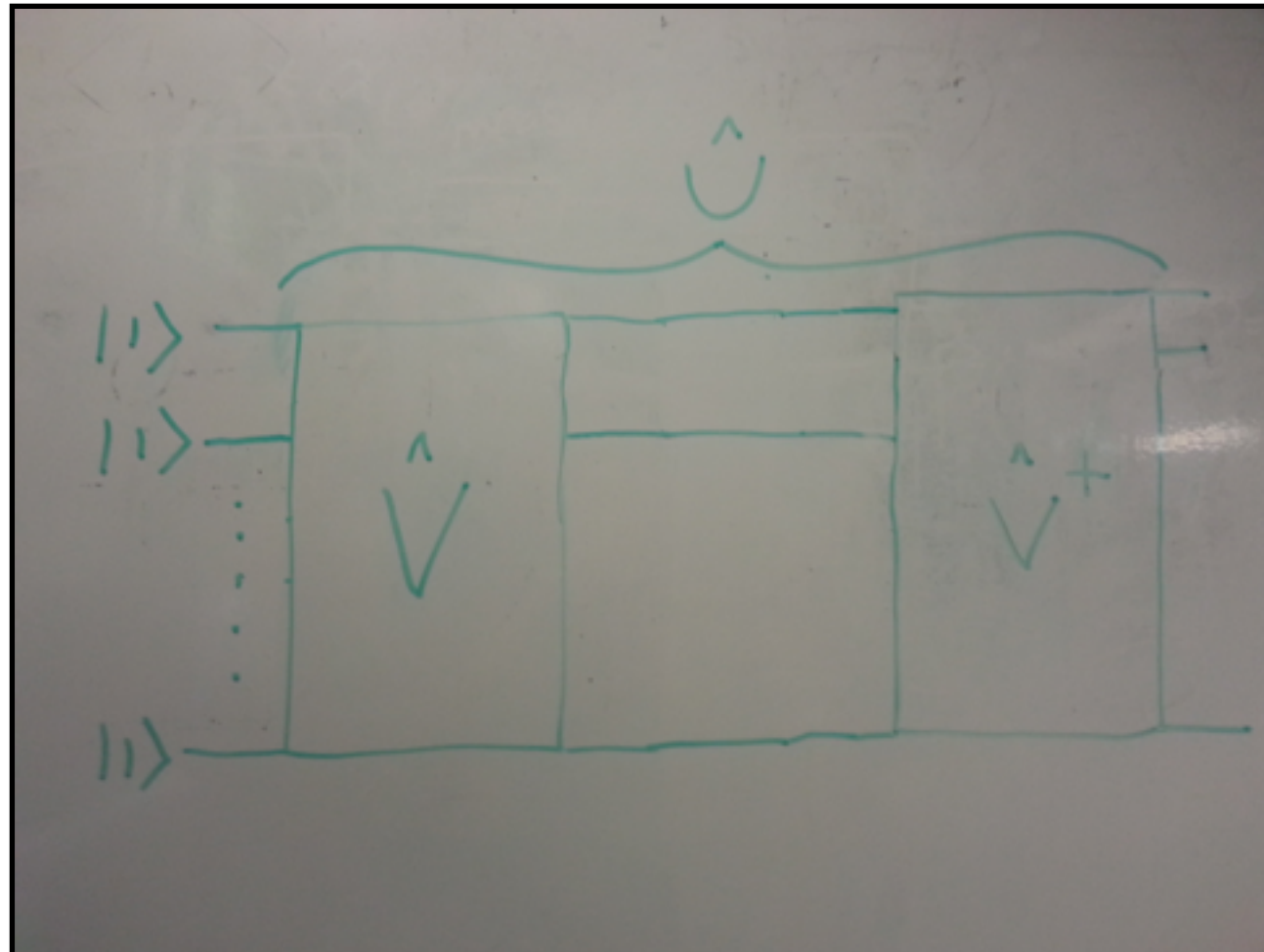


BRAINSTORMING

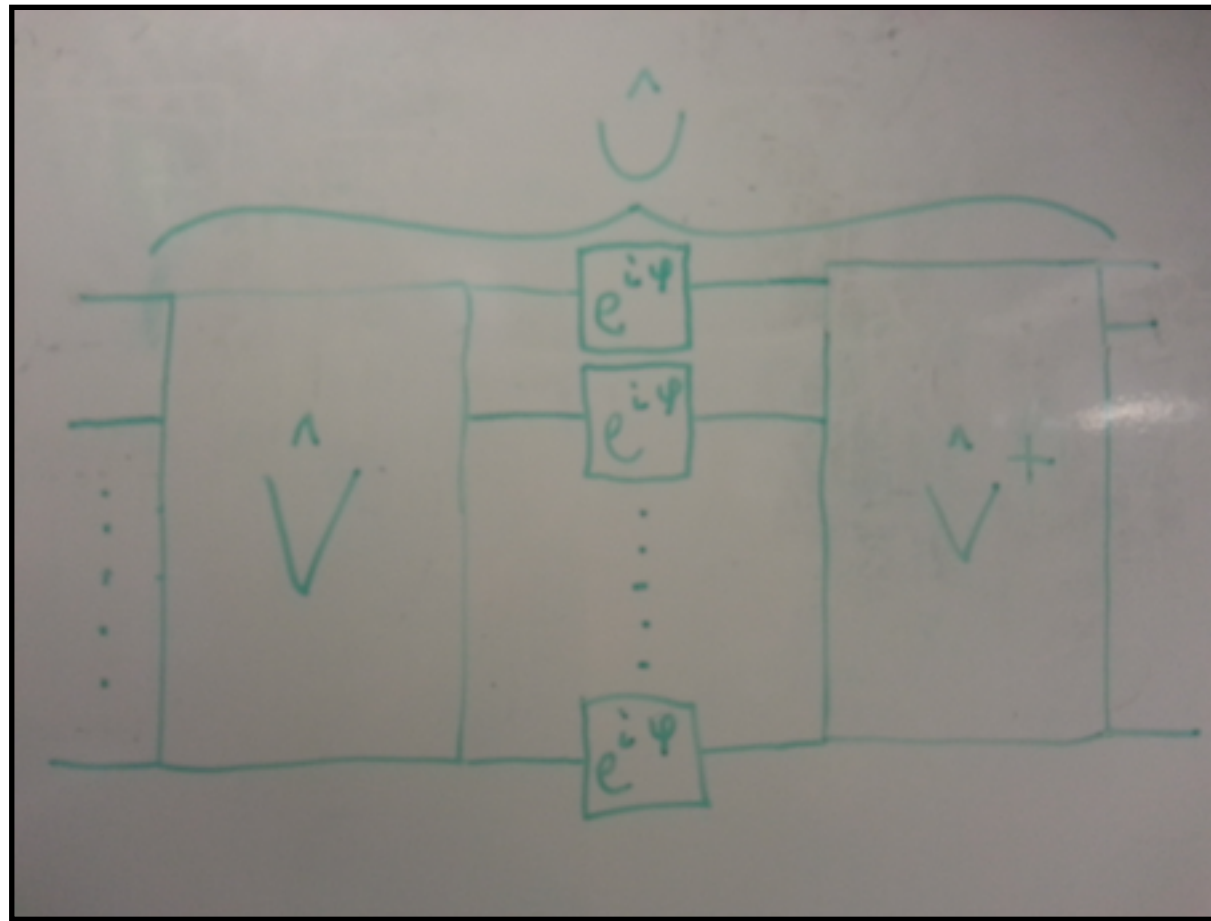
To the laboratory!



METROLOGY WITH BOSON-SAMPLING



METROLOGY WITH BOSON-SAMPLING

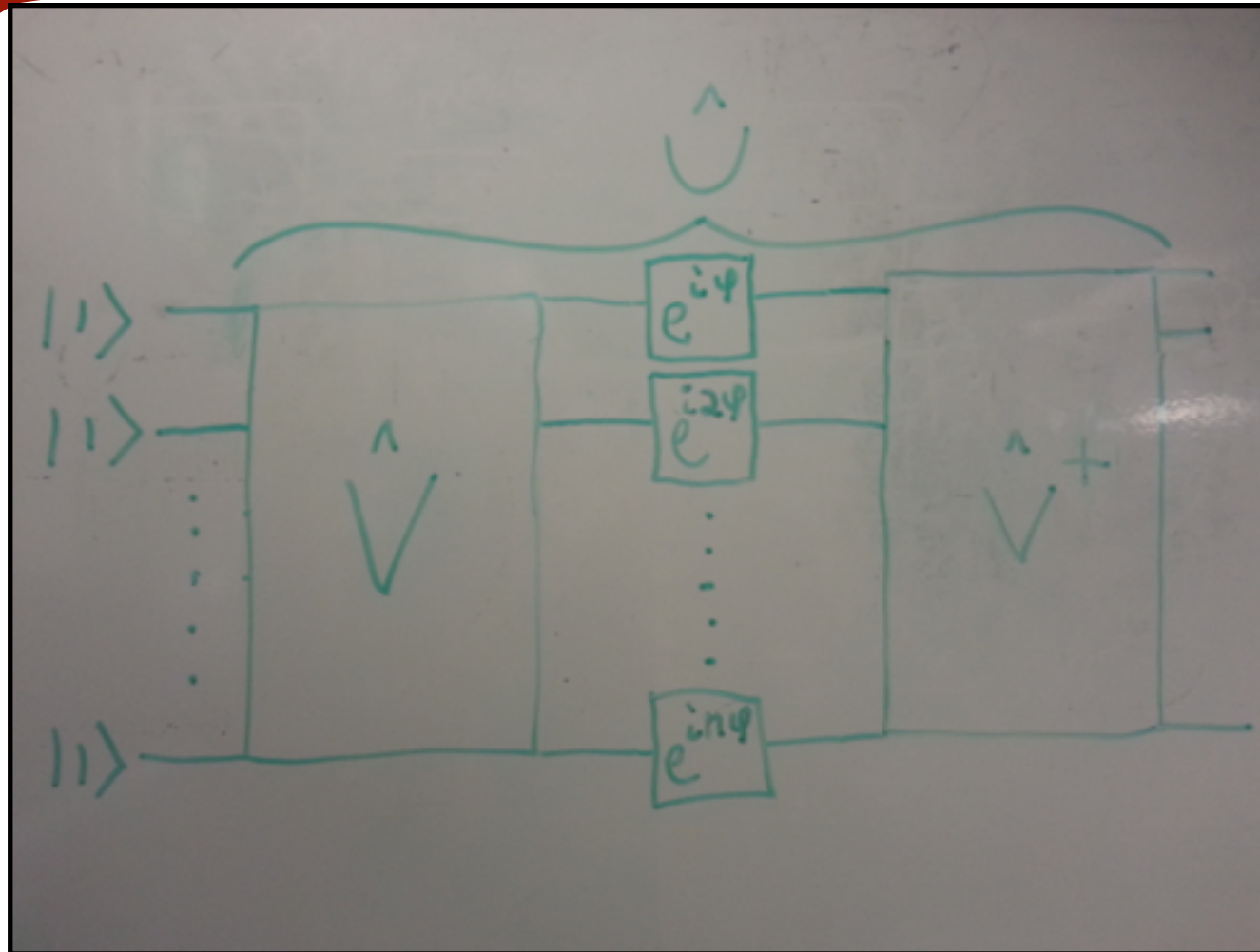


METROLOGY WITH BOSON-SAMPLING

No, No...This is just a global phase and can be factored out.



METROLOGY WITH BOSON-SAMPLING

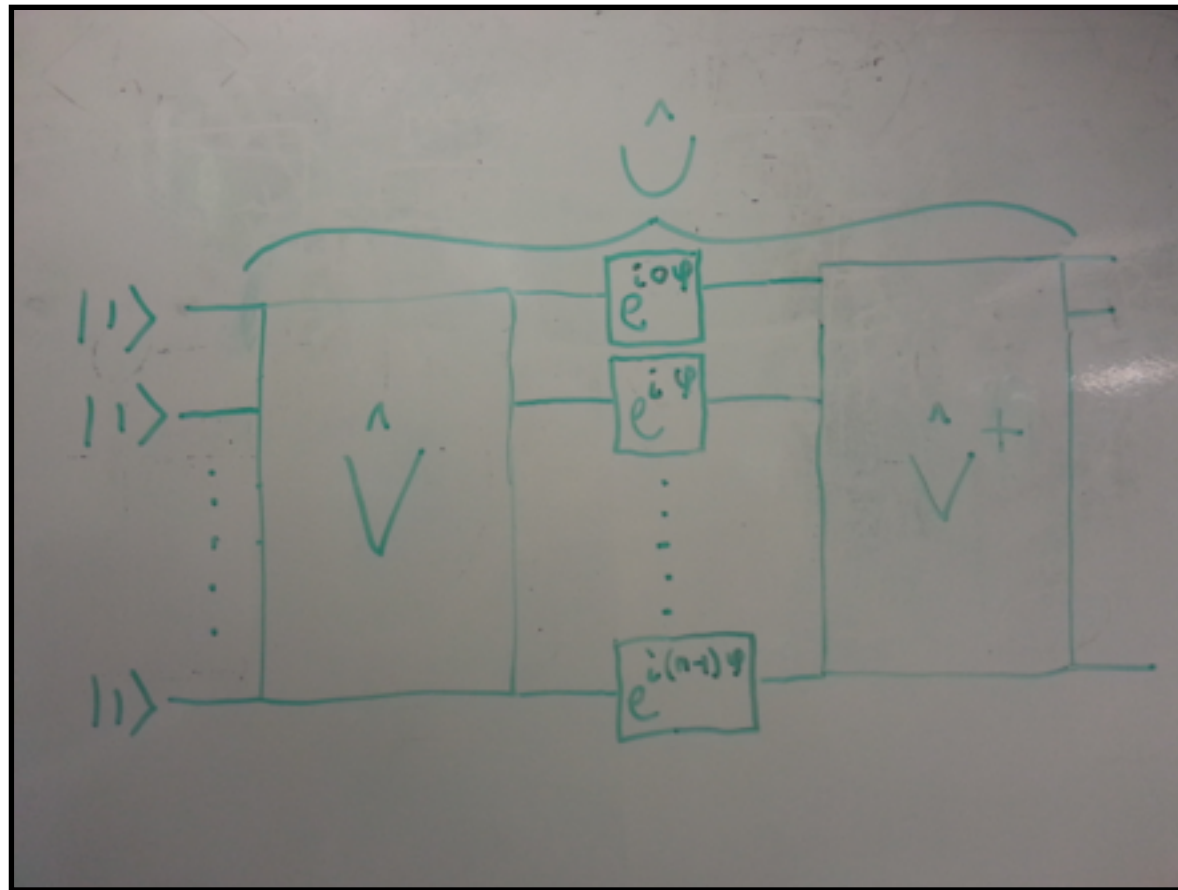


METROLOGY WITH BOSON-SAMPLING

Jon...there is still a global phase...Let's try this



METROLOGY WITH BOSON-SAMPLING



METROLOGY WITH BOSON-SAMPLING

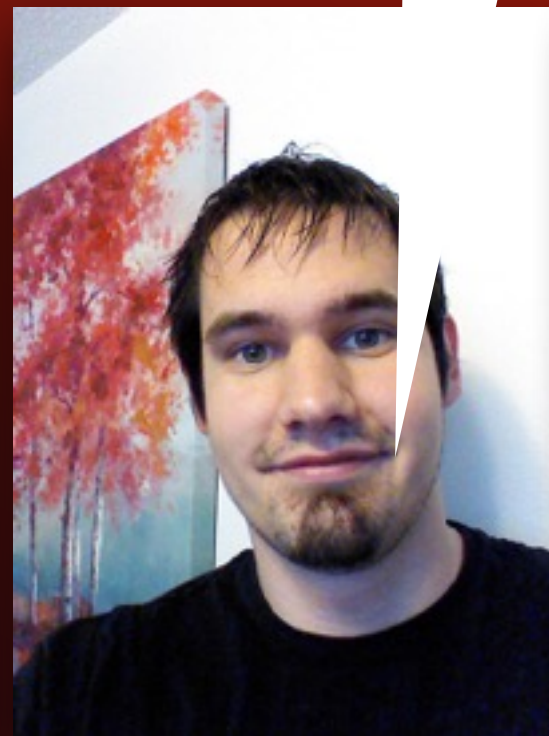
To analyse we need to choose a V .



METROLOGY WITH BOSON-SAMPLING

$$\hat{V} = \hat{H}^{\otimes n}$$

Dimension goes as
 2^n



METROLOGY WITH BOSON-SAMPLING

QFT

$$\hat{V}_{j,k}^{(n)} = \frac{1}{\sqrt{n}} e^{-\frac{2ijK\pi}{n}}$$

Dimension goes as
 n



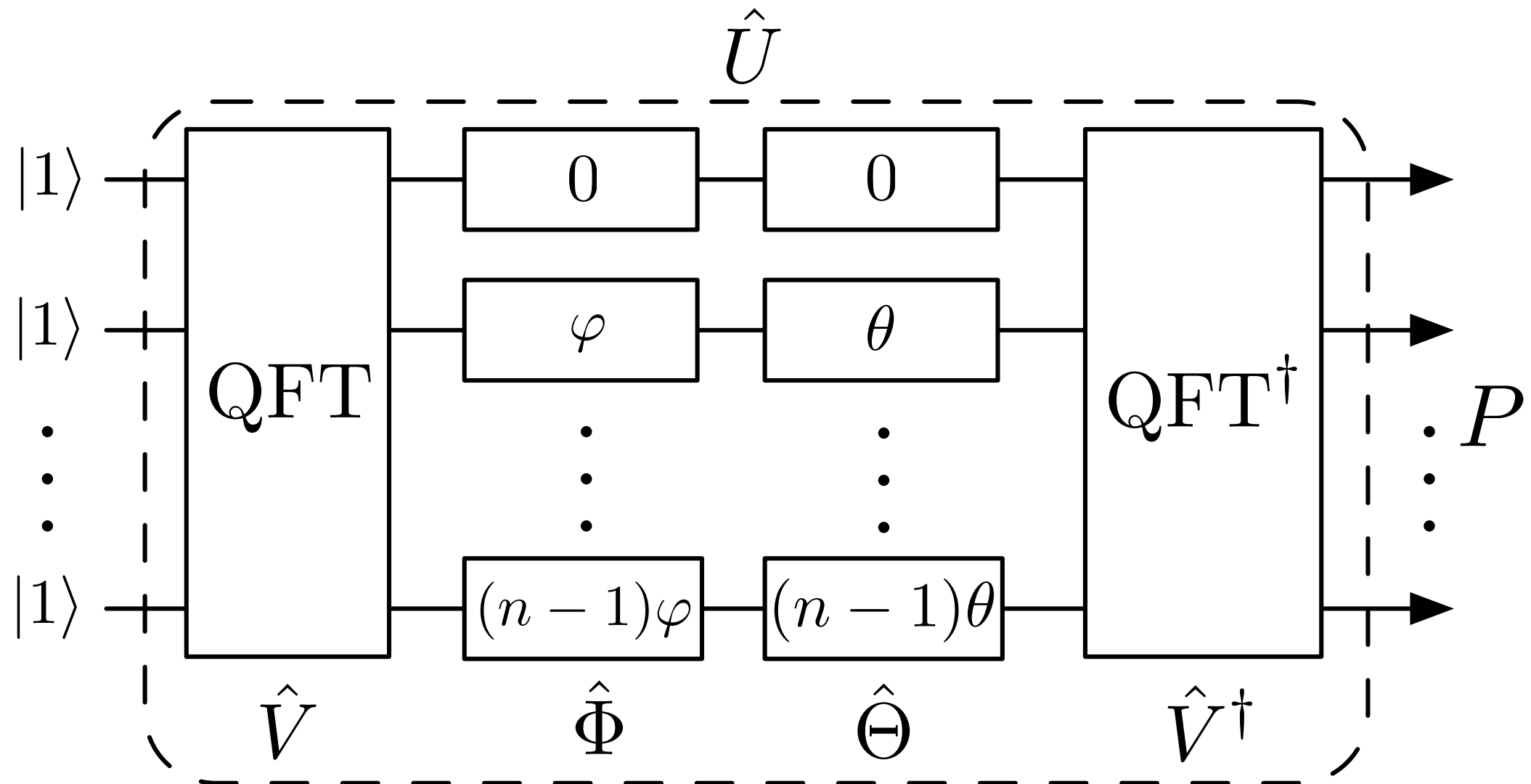
METROLOGY WITH BOSON-SAMPLING

Jonny &
Uruk

Peter you can be
Sauramon



QUANTUM FOURIER TRANSFORM INTERFEROMETER (QUFTI)



$$\hat{U} = \hat{V} \cdot \hat{\Phi} \cdot \hat{\Theta} \cdot \hat{V}^\dagger$$



OBTAINING THE SIGNAL P

$$\hat{U} = \hat{V} \cdot \hat{\Phi} \cdot \hat{V}^\dagger$$

$$V_{j,k}^{(n)} = \frac{1}{\sqrt{n}} \exp \left[\frac{-2ijk\pi}{n} \right] \quad \Phi_{j,k} = \delta_{j,k} \exp \left[i(j-1)\varphi \right]$$

$$U_{j,k}^{(n)} = \frac{1 - e^{in\varphi}}{n(e^{2i\pi(j-k)/n} - e^{i\varphi})}$$

$$P = |\text{Per}(U)|^2$$



OBTAINING THE SIGNAL P

n	$\text{Per}(\hat{U}^{(n)})$
1	1
2	$e^{i\phi} \cos(\phi)$
3	$\frac{1}{9} (2 + e^{3i\phi}) (1 + 2e^{3i\phi})$
4	$\frac{1}{32} (1 + e^{4i\phi}) (3 + e^{4i\phi}) (1 + 3e^{4i\phi})$
5	$\frac{1}{625} (4 + e^{5i\phi}) (3 + 2e^{5i\phi}) (2 + 3e^{5i\phi}) (1 + 4e^{5i\phi})$
6	$\frac{1}{648} (1 + e^{6i\phi}) (2 + e^{6i\phi}) (5 + e^{6i\phi}) (1 + 2e^{6i\phi}) (1 + 5e^{6i\phi})$



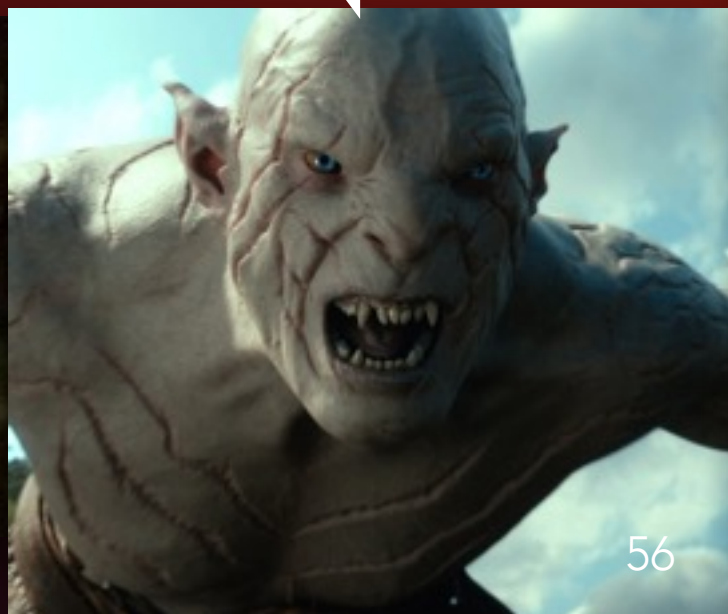
METROLOGY WITH BOSON-SAMPLING

???

???

I got it!

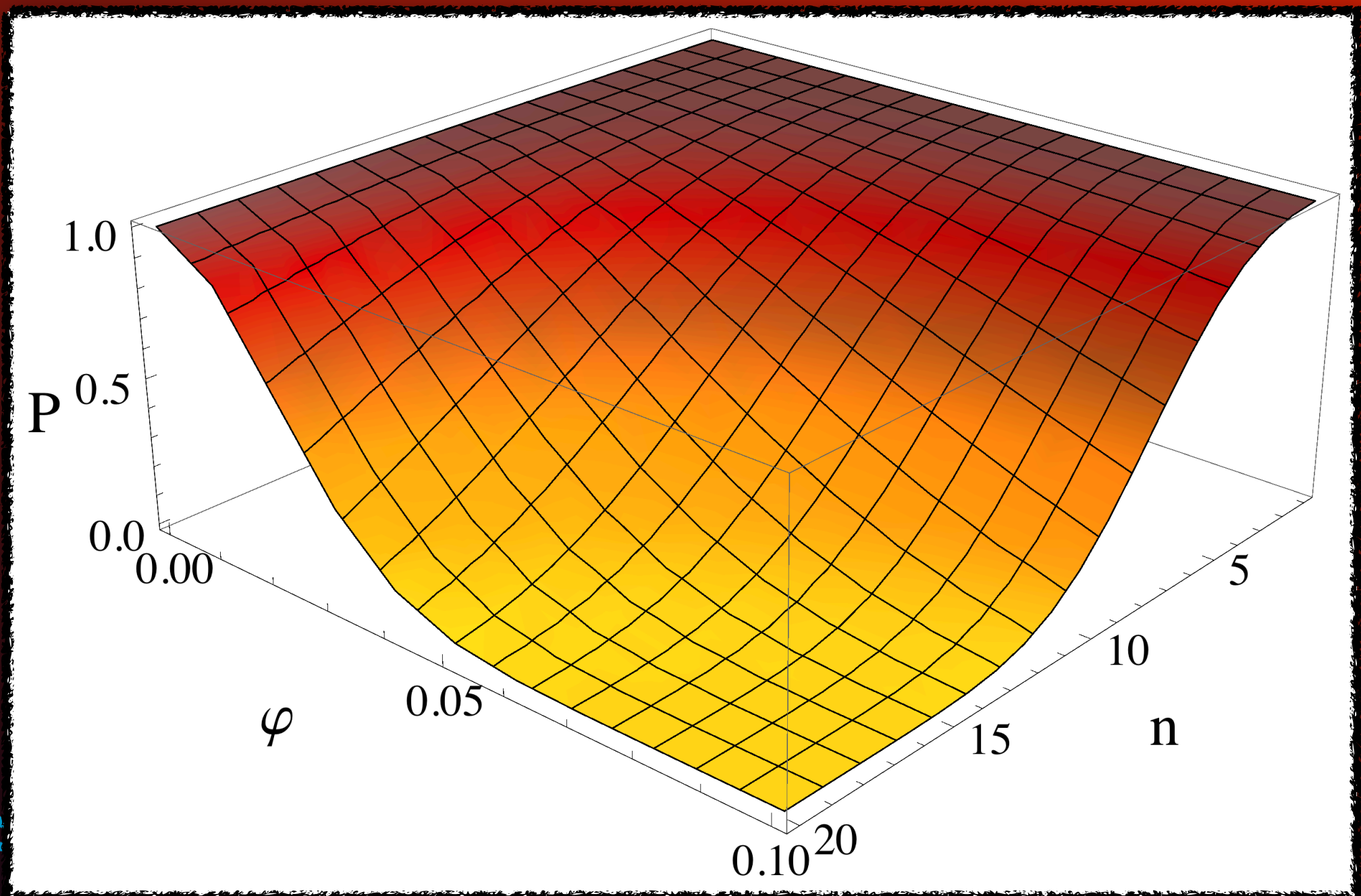
???



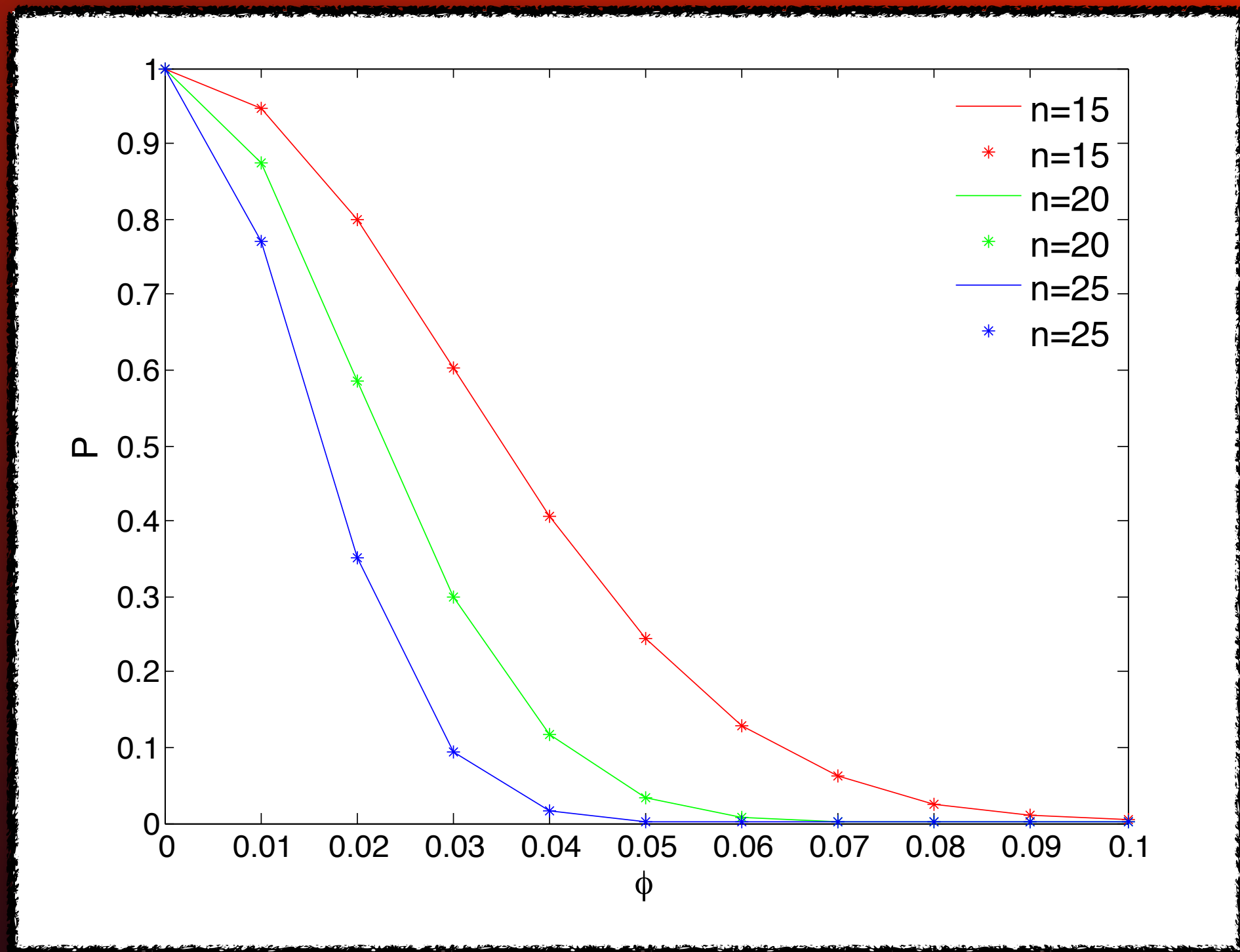
OBTAINING THE SIGNAL P

$$P = |\text{Per}(U)|^2$$

$$P = \frac{1}{n^{2n-2}} \prod_{j=1}^{n-1} \left[a_n(j) \cos(n\varphi) + b_n(j) \right]$$



CONFIRMED TO N=25



OBTAINING P EXPERIMENTALLY

$$P = \frac{x}{x + y}$$

x = # of events that were all 1's

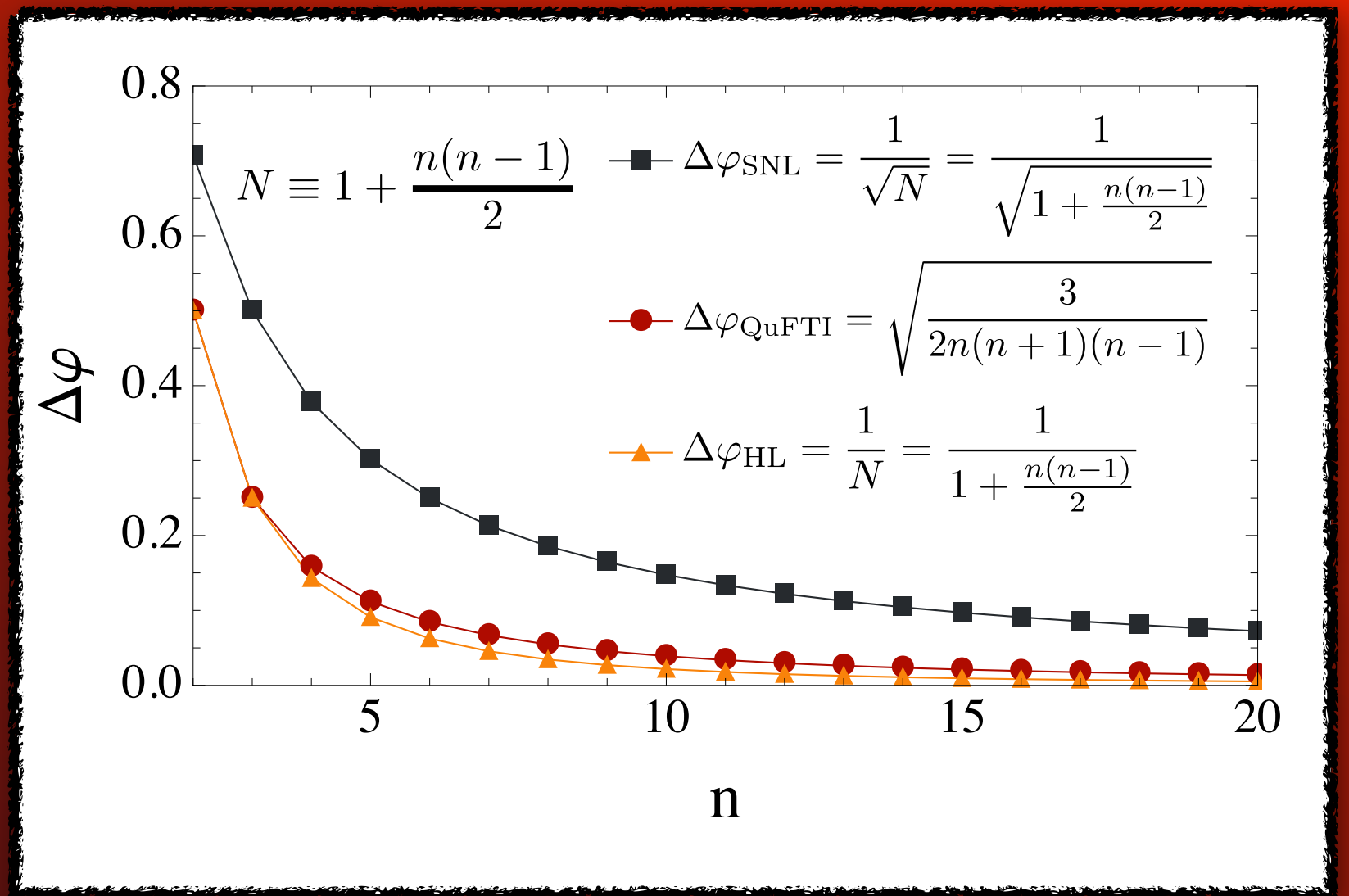
y = # of events that were not all 1's



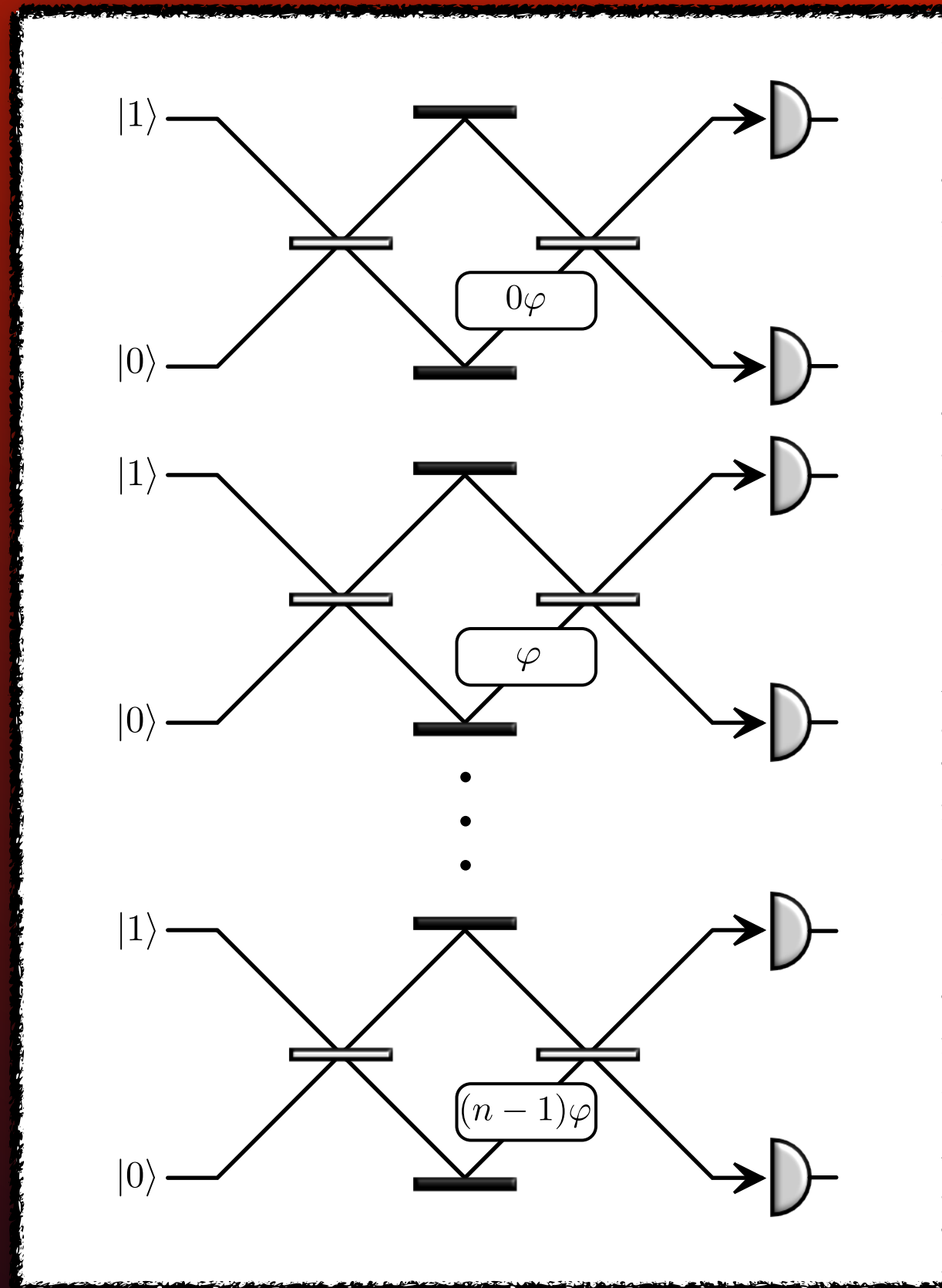
PHASE SENSITIVITY

$$\Delta\varphi = \frac{\sqrt{\langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2}}{\left|\frac{\partial\langle\hat{O}\rangle}{\partial\varphi}\right|}$$

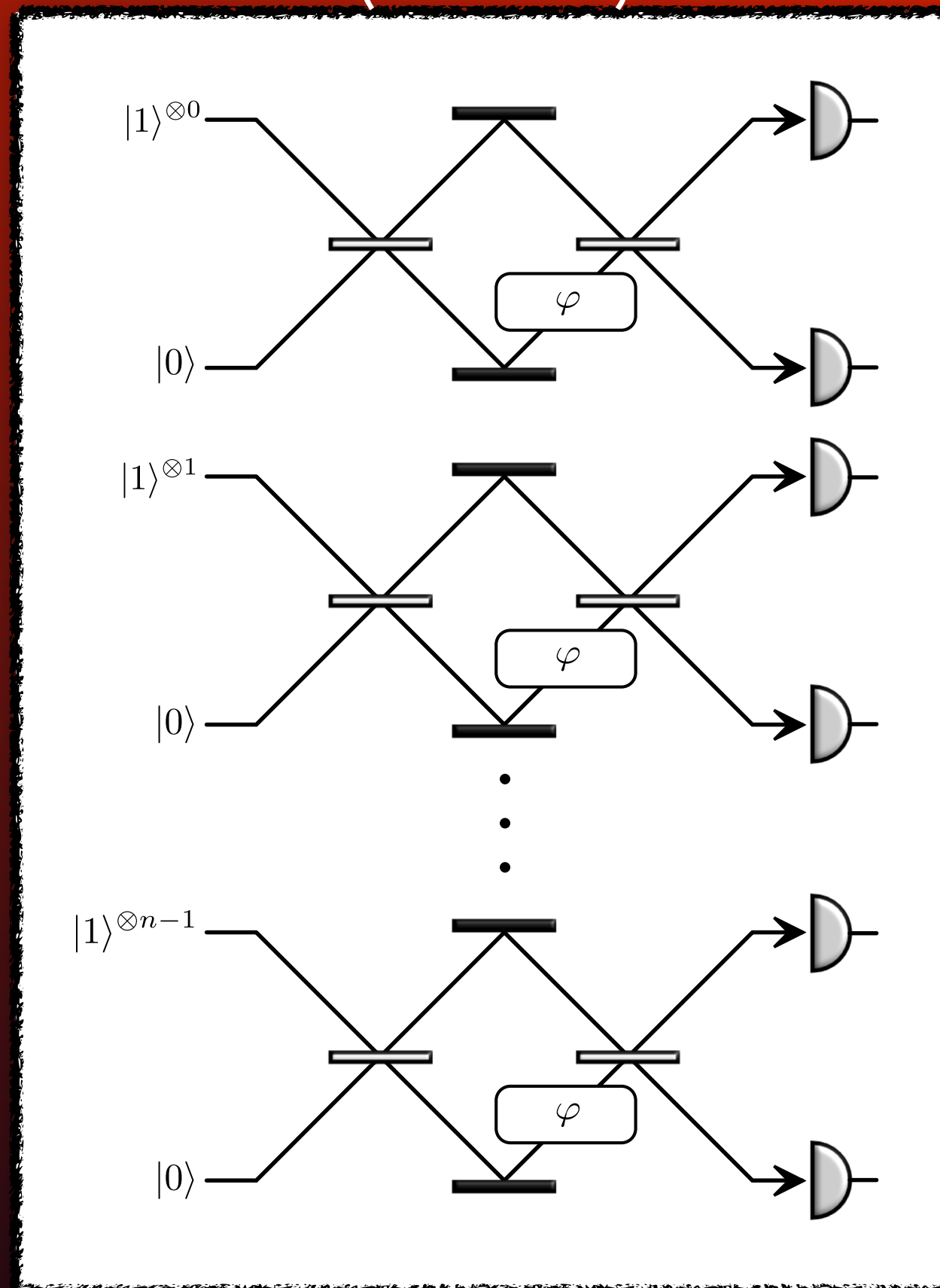
$$\Delta\varphi = \frac{\sqrt{P - P^2}}{\left|\frac{\partial P}{\partial\varphi}\right|}$$



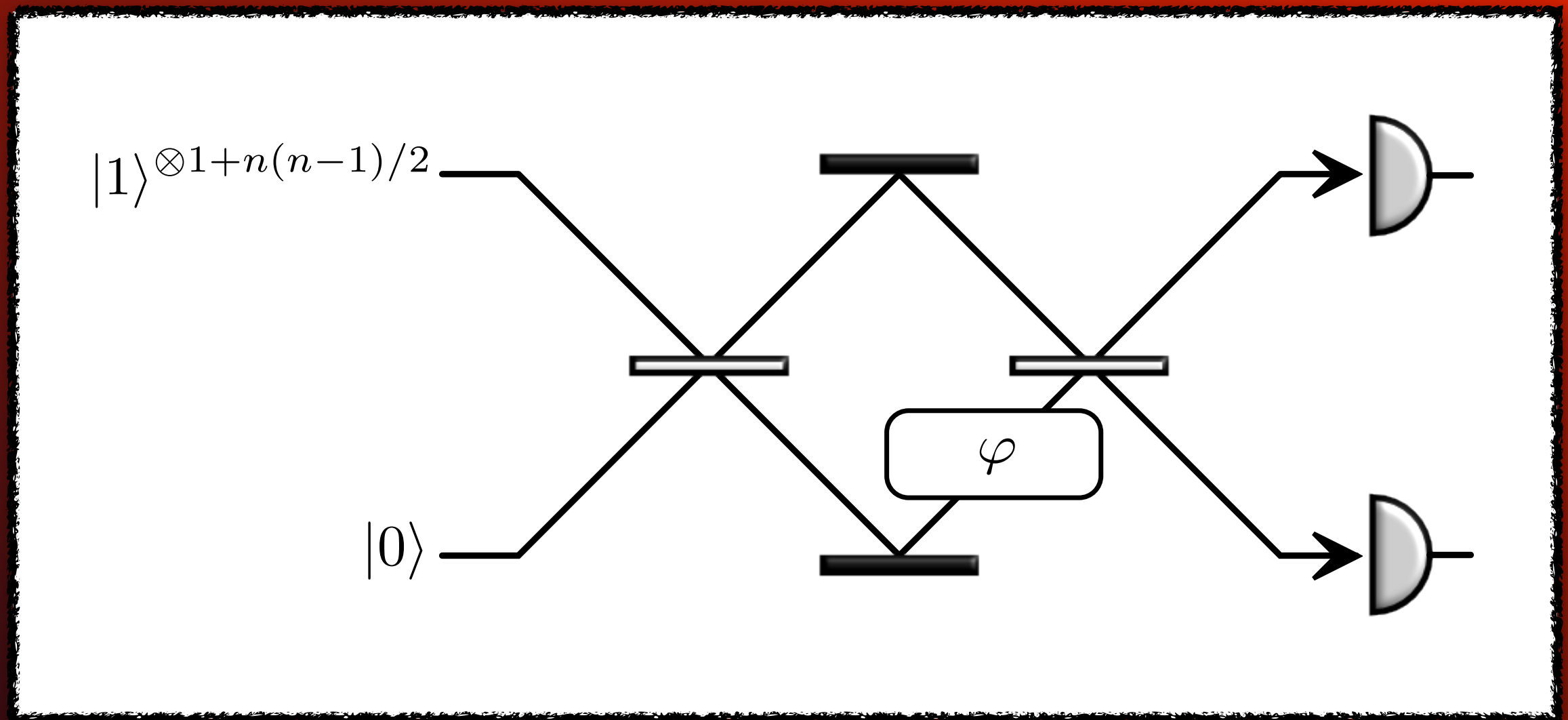
ORDINAL RESOURCE COUNTING (ORC)



ORDINAL RESOURCE COUNTING (ORC)



ORDINAL RESOURCE COUNTING (ORC)



$$N \equiv 1 + \frac{n(n-1)}{2}$$



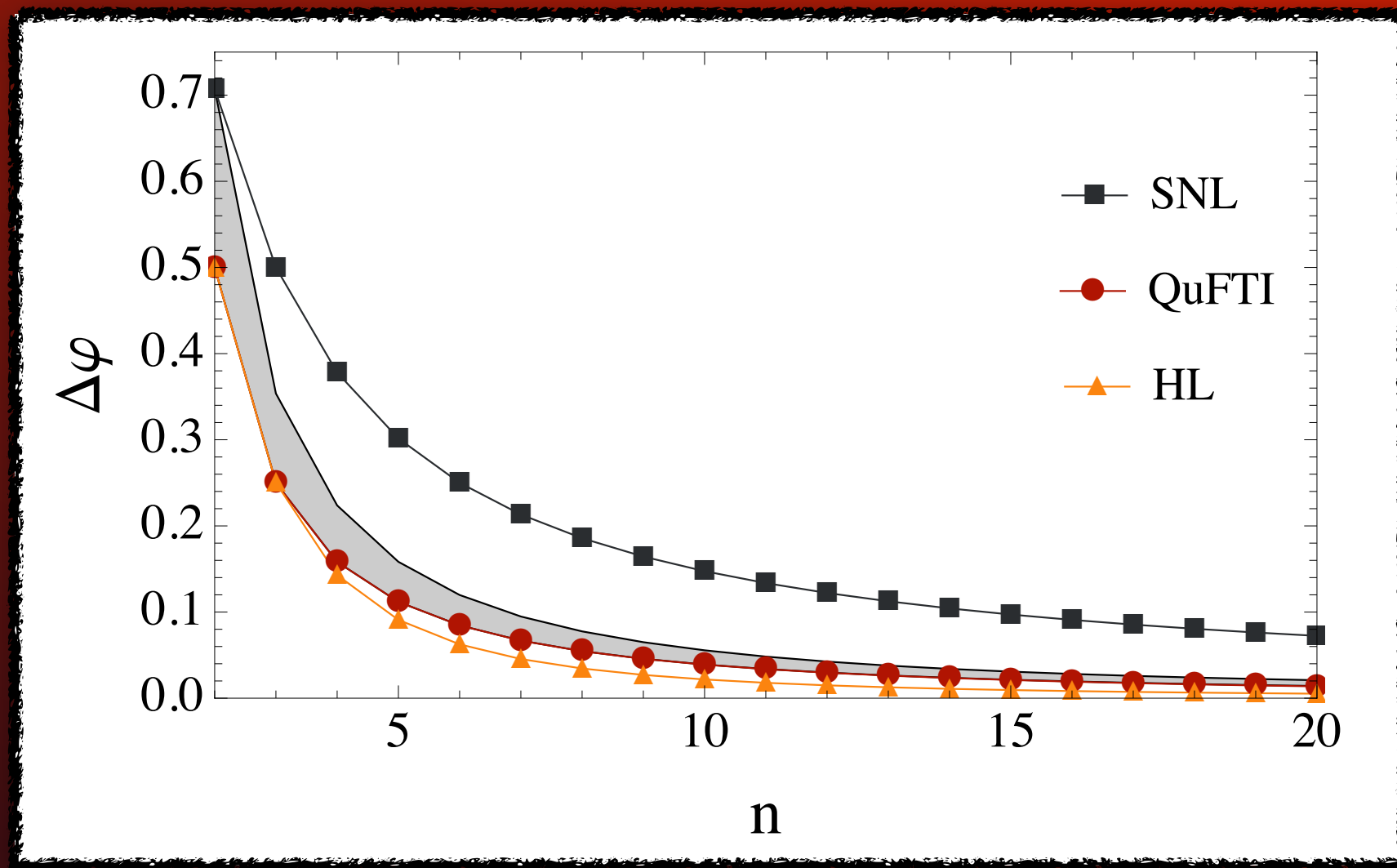
DEPHASING IN MORDOR

$$\begin{aligned} e^{\pm ij\varphi} &\rightarrow e^{\pm ij(\varphi+\Delta\chi)} \\ &= e^{\pm ij\varphi} e^{\pm ij\Delta\chi} \\ &= e^{\pm ij\varphi} \left(1 \pm ij \underbrace{\Delta\chi}_{=0} - \frac{1}{2} j \Delta\chi^2 \pm \dots \right) \\ &= e^{\pm ij\varphi} \left(1 - \frac{1}{2} j \Delta\chi^2 \pm \dots \right) \\ &\approx e^{\pm ij\varphi} e^{-\frac{1}{2} j^2 \Delta\chi^2} \end{aligned}$$

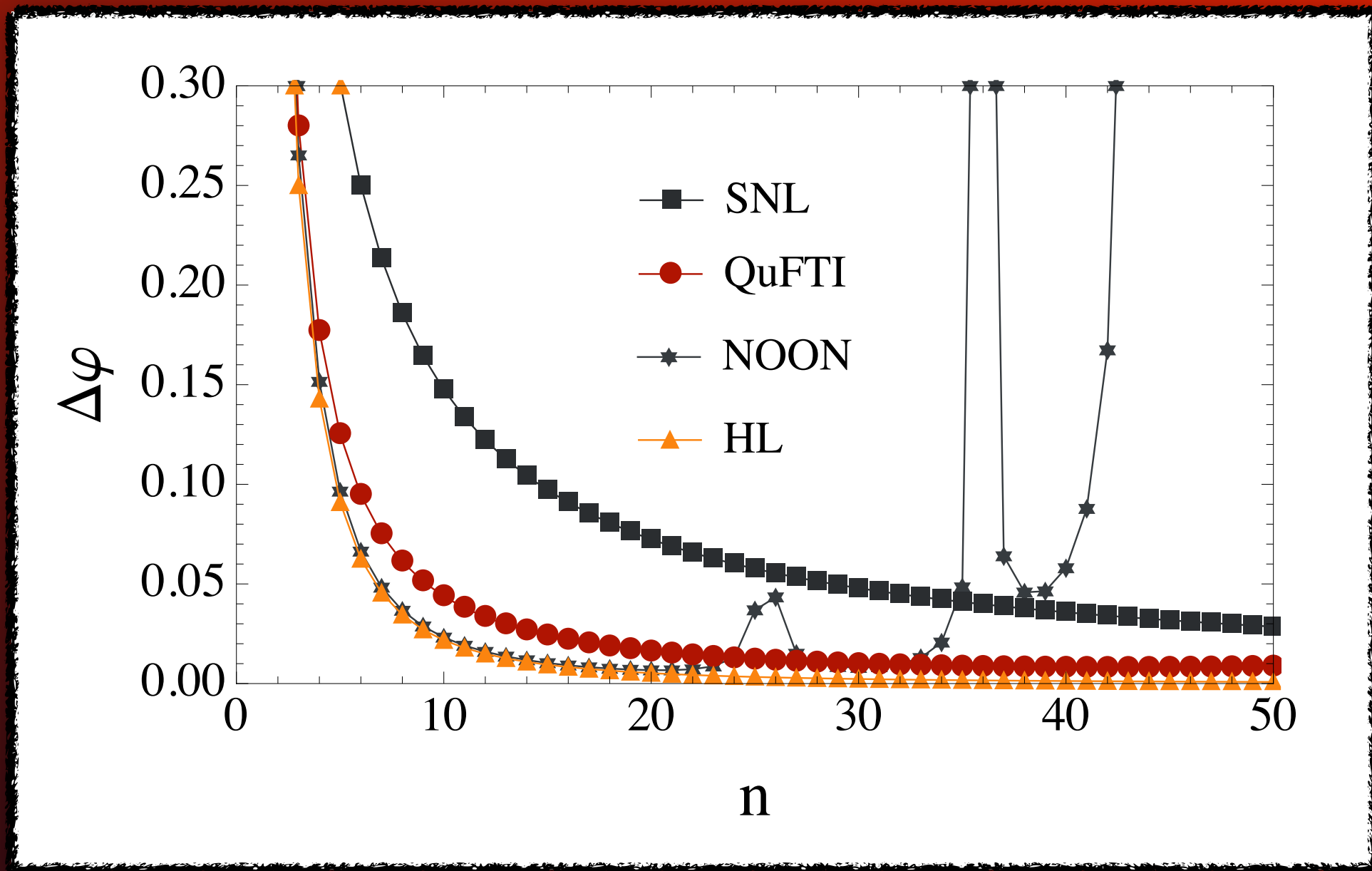


DEPHASING IN MORDOR

$$e^{\pm i j \varphi} e^{-\frac{1}{2} j^2 \Delta \chi^2}$$



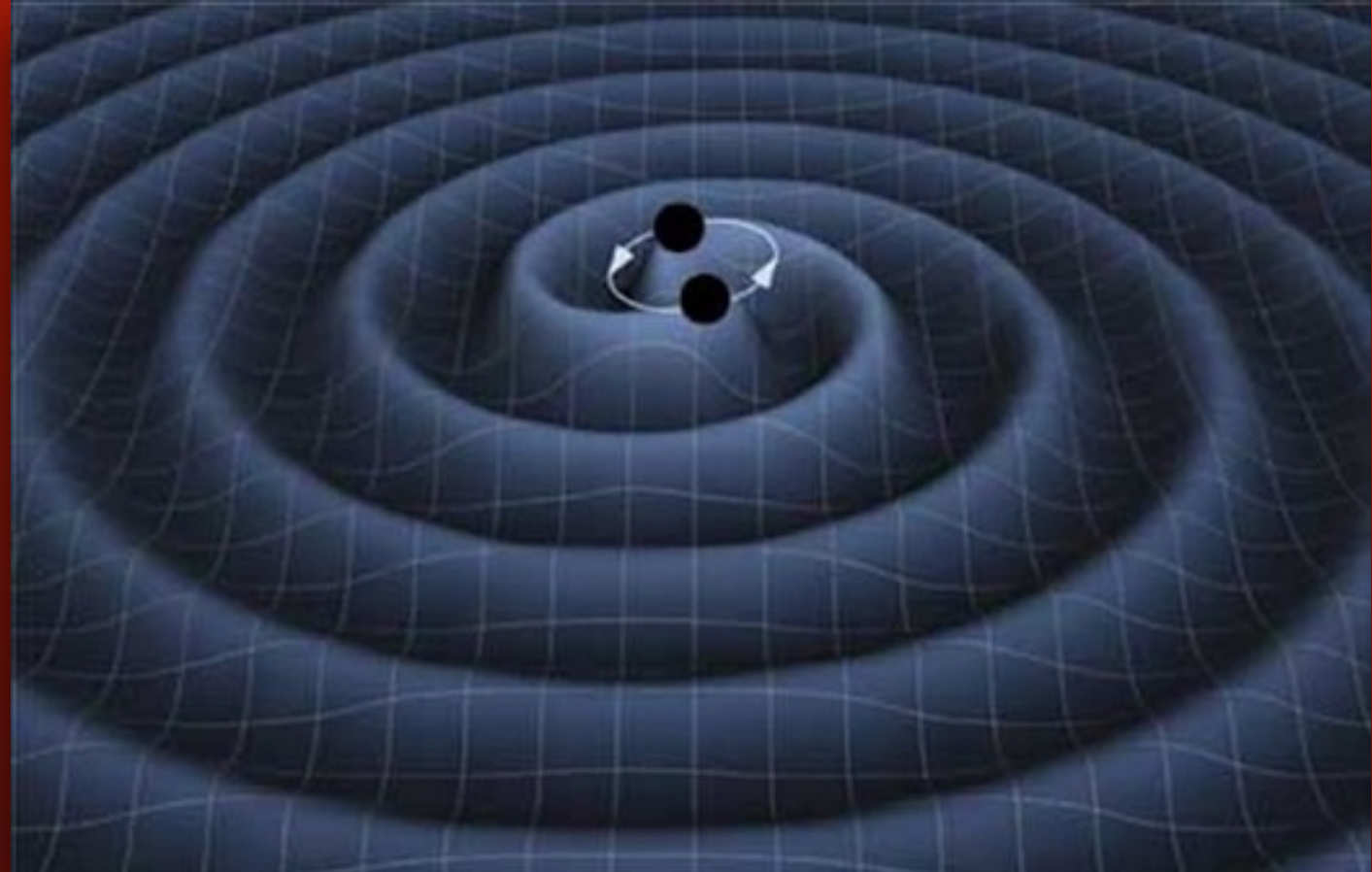
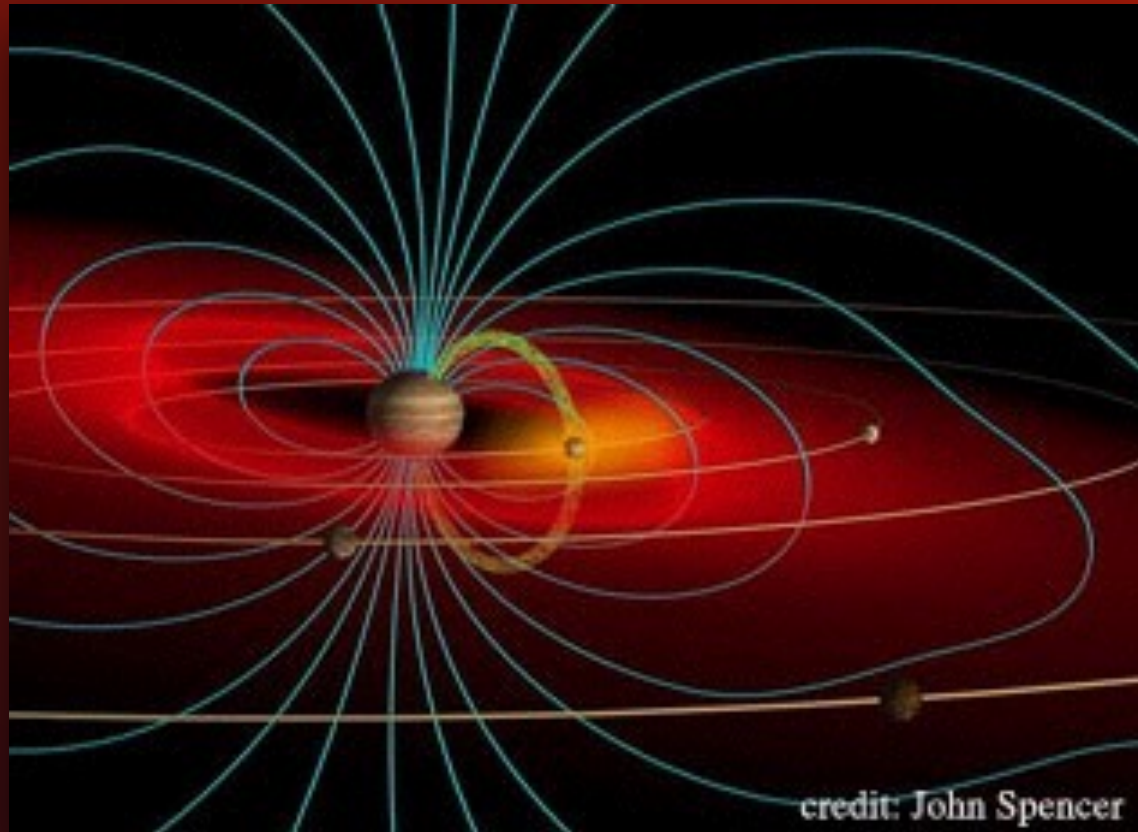
DEPHASING IN MORDOR



$$\varphi = 0.01, \chi = 0.005$$



WHAT WOULD WE MEASURE



FINAL REMARKS

OTHER CONFIGURATIONS

NON-LINEAR PHASE GRADIENT

DIFFERENT INTERFEROMETER FOR V

DIFFERENT INPUT STATE

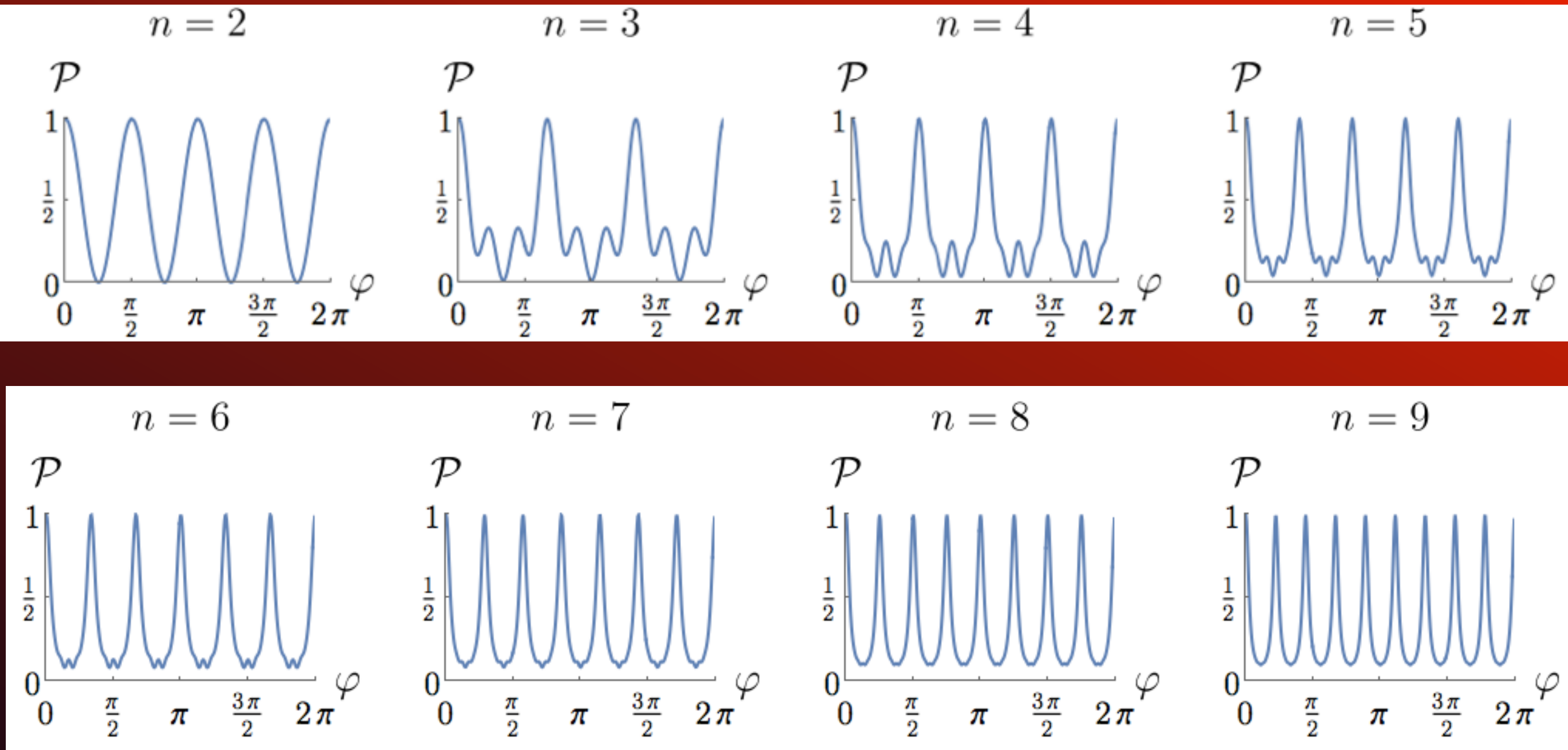
MORE QUANTUM TECHNOLOGIES

LITHOGRAPHY

IMAGING



OTHER RESEARCH: QUANTUM IMAGING



CONCLUSION

THANK
YOU!

