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## Metastable Regimes: a Parametric Study in Reference to Single-Phase Parallel Channel Natural Circulation Systems

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### Abstract

The objective of the present study is to predict the existence of metastable regime (multiple steady states) and effects of various parameters on it, in a parallel channel system under natural circulation. To examine the basic behavior of such a system, a three parallel channel system with non-uniform heat inputs and a common downcomer, was investigated in detail both analytically and using RELAP5/ Mod 3.2 computer code. The results showed the existence of a metastable regime for a given set of heat input ratio. A study was also performed to investigate the effect of diameter, channel height and orificing at the inlet of heater, on the metastable regime. A Non-Dimensional Heat Flux Ratio  $(N_H)$  has been proposed to predict the flow reversal. The heat flux level at which flow reversal occurs has been predicted for different rates of power rise and power reduction (with different input heat flux). It was found that the power level at which the flow reversal occurs depends upon the operating procedures (i.e. power rise and power reduction). The flow direction in different channels will depend on the prevailing density difference among channels and downcomer. It was found that, the flow direction in a channel, where power variation is taking place, follows the maximum buoyancy hydraulic path.

**Keywords**: metastable regime, parallel channel systems, flow reversal

### NOMENCLATURE

#### **General Symbols**

| Α     | : flow area, $m^2$                     |
|-------|--|
| a, b  | : constant in friction factor relation |
| $c_p$ | : specific heat, $J / kg K$            |

| Ch          | Channel                                    |
|-------------|--|
| D           | : Channel<br>: hydraulic diameter <i>m</i> |
| f           | : Darcy-Weisbach friction factor           |
| g           | : gravitational acceleration, $m/s^2$      |
| K           | : local pressure loss coefficient          |
| L           | : length, <i>m</i>                         |
| $N_{m}$     | : non-dimensional mass flow rate,          |
|             | $W_{un}/W_h$                               |
| $N_{\rm H}$ | : non-dimensional heat flux ratio,         |
|             | $q_{un}^{"}/q_{h}^{"}$                     |
| Р           | : pressure, $N/m^2$                        |
| q           | : power supplied, $W$                      |
| q''         | : Heat flux, $W/m^2$                       |
| Q           | : non-dimensional power defined in         |
|             | equation (19)                              |
| Re          | : Reynolds number, $DW/A\mu$               |
| $R_q$       | : non-dimensional power ratio              |
| Т           | : temperature, $K$                         |
| W           | : mass flow rate, $kg/s$                   |
| z           | : elevation, <i>m</i>                      |
| Crea        | Ir Symbola                                 |

### **Greek Symbols**

| β                   | : thermal expansion coefficient, $K^{-1}$ |  |  |
|---------------------|---|--|--|
| ρ                   | : density, $kg/m^3$                       |  |  |
| $\Delta  ho$        | : density difference, $kg/m^3$            |  |  |
| $\overline{ ho}$    | : average density, $kg/m^3$               |  |  |
| ξ                   | : coefficient in dynamic viscosity        |  |  |
| variation, $K^{-1}$ |   |  |  |

### Subscripts

DC : downcomer

- *eq* : equivalent
- *eff* : effective
- *h* : heated channel
- *m* : mean
- *n* : number of channels
- *un* : unheated channel
- 0 : reference quantity
- 1,2,3 : channel number

## Introduction

Natural circulation flows in multiple-channel systems operating with various levels of heat inputs occur in diverse applications. One of these can be found in nuclear reactors where under certain normal operating or upset conditions, natural circulation flow cools the core. Unequally heated parallel channel systems can exhibit interesting flow behaviours during natural circulation. For example, the density difference between the downcomer and any channel leads to a buoyancy force favoring upward flow in the heated channel. On the other hand, the density difference between two unequally heated channels favors downward flow in the low power channel. The actual flow direction will depend on the larger of the two buoyancy forces. Also, it may lead to the existence of a metastable regime in parallel channel systems with non-uniform heat inputs. By definition, a system is in a state of metastable equilibrium if it is stable to small disturbances but not to large disturbances. Hence it is of particular importance to predict the thresholds of the metastable regime, so that this can be taken care of in the design.

To our knowledge, Chato [1] was the first to study, analytically and experimentally, the metastable regime in a three channel system with one channel heated, one channel unheated, and the third channel either heated or cooled. For this arrangement he found that two different stable flow rates can occur for a given set of channel heating rates. He has predicted the flow reversal of a heated channel from down flow to up flow while increasing power of the down flowing channel, keeping the power of the heated upward flowing channel constant. But the range of power level at which metastable regime occurs has not been clearly shown. Zvirin [2] has theoretically studied the stability of various steady state flows in a thermosyphon with multiple vertical channels. He has found a modified Rayleigh

number to describe the onset of flows and instabilities. Takeda et al. [3] numerically and experimentally studied a system of four vertical channels with different heat generation rate. They found that a one-dimensional modeling of the channels could predict the reversal of a heated channel from downflow to upflow, but not from upflow to downflow. Yahalom et al. [4] has suggested a downflow preference number, which says that a heated channel with the highest value of non-dimensional preference number will flow downward. Todreas et al. [5] defined an upflow preference number. This says that in a parallel channel system in which the heated channels are in downflow, the channel with the highest upflow preference number will reverse first from downflow to upflow. Preference numbers can only tell which channels are susceptible to flow reversal, but the question that still remain is exactly at what power or heat flux ratio the flow in a heated channel will reverse from upflow to downflow and vice versa. Does rate of increment or decrement of power affect the power level at which flow reverses? Therefore, an analysis has been carried out to predict the metastable regime in a natural circulation system with three parallel vertical channels connected between common inlet and outlet headers and a downcomer.

In the present analysis, the possible existence of hysteresis in the natural circulation loop has been investigated in detail both analytically and using RELAP5/ Mod 3.2. The results showed that multiple steady states differing in flow direction may occur for certain heat flux. The region bounded by this hysteresis is the metastable regime as discussed by Chato (1963). The analysis also showed that the hysteresis regime depends on the rate of reduction or increase of power emphasizing the importance of operating procedure to avoid flow reversal. The possibility of the existence of the metastable regime has been investigated with nonuniform heat inputs with RELAP5/ Mod 3.2 [6] computer code.

### **Mathematical Formulation**

Assuming one dimensional approximation to be valid, the conservation equations for the geometry given in Fig. 1 can be written as

**Momentum equation:** Steady state momentum equation for a channel 'n' at an axial location z can be written as,

$$\left(-\frac{dP}{dz}\right) = \rho g + \frac{f}{D_n} \frac{W_n |W_n|}{2\rho A^2} + K \frac{W_n |W_n|}{2\rho A^2} \frac{1}{dz}$$
(1)

(1/dz) factor says that local form losses are at discrete points and are not continuous. This factor disappears upon integrating along the channel length.



Fig. 1: Schematic of Parallel Channel Natural Circulation Loop

**Energy equation:** 

$$q_n = W_n c_p \Delta T_{overall} \tag{2}$$

**Continuity equation:** 
$$\sum W_n = 0$$
 (3)

Integrating equation (1) from bottom to top results

$$-\int_{0}^{L} \frac{dP}{dz} dz = g \int_{0}^{L} \rho \, dz + \sum_{Z=0}^{Z=L} \left( \frac{fL}{D} + K \right) \frac{W_n |W_n|}{2 \, \rho \, A^2} \tag{4}$$

Assuming Boussinesq approximation to be valid, the density in the heated channel can be calculated as

 $\rho = \rho_0 (1 - \beta \Delta T) = \rho_0 [1 - \beta (T - T_0)]$ 

Now, the buoyancy term in equation (4) can be calculated as

$$\int_{0}^{L} \rho \, dz = \int_{0}^{L} \rho_0 \big( 1 - \beta \, \Delta T \big) dz$$

Assuming linear variation of temperature within the heated section and q to be uniform,

 $\Delta T = \frac{z}{L} (\Delta T)_{overall}; \text{ hence the buoyancy term}$ 

will become

$$\int_{0}^{L} \rho \, dz = \rho_0 L \left[ 1 - \frac{\beta}{2} \Delta T_{overall} \right]$$

$$\rho_0 \left[ 1 - \frac{\beta}{2} \Delta T_{overall} \right] = \frac{\int_0^L \rho \, dz}{L} = \rho_m \text{ and}$$
$$\Delta T_{overall} = \frac{2}{\beta} \left( 1 - \frac{\rho_m}{\rho_0} \right) \tag{5}$$

From equation (4)

$$\begin{bmatrix} \Delta P \end{bmatrix}_{Upper Plenum}^{Lower Plenum} = g\rho_0 L \begin{bmatrix} 1 - \frac{\beta}{2} \Delta T_{overall} \end{bmatrix} + \sum \left( \frac{fL_{eff}}{D} \right) \frac{W_n |W_n|}{2\rho A^2}$$
(6a)

Assuming the variation of friction factor as

$$f = \frac{a}{\text{Re}^b}$$
 where the local Reynolds

number  $\operatorname{Re} = \frac{D_n W_n}{A_n \mu}$ . Thus, equation (6a) becomes

$$\begin{bmatrix} \Delta P \end{bmatrix}_{Upper Plenum}^{Lower Plenum} = g\rho_0 L \begin{bmatrix} 1 - \frac{\beta}{2} \frac{2}{\beta} \left( 1 - \frac{\rho_m}{\rho_0} \right) \end{bmatrix}$$
$$+ \frac{a L_{eff} (W_n)^{1-b} |W_n|}{2A^{2-b} D^{1+b}} \int_0^L \frac{\mu^b}{\rho} dz$$

(6b)

Now, assuming a linear variation of dynamic viscosity with temperature in side the heated

channel, 
$$\int_{0}^{L} \mu^{b} dz = \mu_{0}^{b} L \left[ 1 - \frac{\xi}{2} \Delta T_{overall} \right]$$
$$\left[ \Delta P \right]_{Upper Plenum}^{Lower Plenum} = g \rho_{0} L \left[ \frac{\rho_{m}}{\rho_{0}} \right]$$
$$+ C_{n} (W_{n})^{1-b} |W_{n}| \left[ 1 - \frac{\xi}{\beta} \left( 1 - \frac{\rho_{m}}{\rho_{0}} \right) \right]$$
(7)  
where  $C = \frac{a L_{eff} \mu_{0}^{b} L}{2 A^{2-b} D^{1+b} \rho_{0}}$ ,  $L_{eff} = L_{i} + L_{eq}$ ,
$$L_{eq} = K \left( \frac{D}{f} \right), \ \beta = \frac{1}{\rho_{0}} \left( \frac{\partial \rho}{\partial T} \right)_{P=const}$$
$$\xi = \frac{1}{\mu_{0}} \left( \frac{\partial \mu}{\partial T} \right)_{P=const}$$

### **Condition for Metastable Regime**

Without loss of generality, the condition for possible metastable flow regime has been derived for a two parallel heated channels with a common downcomer system as follows: For Channel 1:

$$\Delta P_{1} = g \rho_{0} L \left[ \frac{(\rho_{m})_{1}}{\rho_{0}} \right] + C_{1} W_{1}^{1-b} |W_{1}| \left[ 1 - \frac{\xi_{1}}{\beta_{1}} \left\{ 1 - \frac{(\rho_{m})_{1}}{\rho_{0}} \right\} \right]$$
(8)

For Channel 2:

$$\Delta P_{2} = g \rho_{0} L \left[ \frac{(\rho_{m})_{2}}{\rho_{0}} \right] + C_{2} W_{2}^{1-b} |W_{2}| \left[ 1 - \frac{\xi_{2}}{\beta_{2}} \left\{ 1 - \frac{(\rho_{m})_{2}}{\rho_{0}} \right\} \right]$$
(9)

Similarly for Channel 3:

$$\Delta P_{3} = g \rho_{0} L \left[ \frac{(\rho_{m})_{3}}{\rho_{0}} \right] + C_{3} W_{3}^{1-b} |W_{3}| \left[ 1 - \frac{\xi_{3}}{\beta_{3}} \left\{ 1 - \frac{(\rho_{m})_{3}}{\rho_{0}} \right\} \right]$$
(10)

Since, Channel 3 is unheated  $(\rho_m)_3 = \rho_0$ 

Hence,  $\Delta P_3 = g \rho_0 L + C_3 W_3^{1-b} |W_3|$ (11)Putting the boundary condition for parallel channel system that,  $\Delta P_1 = \Delta P_2 = \Delta P_3$  and using equation (11), equation (8) and equation (9) becomes,

$$g \rho_{0} L\left[\frac{(\rho_{m})_{1}}{\rho_{0}}\right] + C_{1} W_{1}^{1-b} |W_{1}| \left[1 - \frac{\xi_{1}}{\beta_{1}} \left\{1 - \frac{(\rho_{m})_{1}}{\rho_{0}}\right\}\right]$$

$$= g \rho_{0} L + C_{3} W_{3}^{1-b} |W_{3}|$$

$$g \rho_{0} L\left[\frac{(\rho_{m})_{2}}{\rho_{0}}\right] + C_{2} W_{2}^{1-b} |W_{2}| \left[1 - \frac{\xi_{2}}{\beta_{2}} \left\{1 - \frac{(\rho_{m})_{2}}{\rho_{0}}\right\}\right]$$

$$= g \rho_{0} L + C_{3} W_{3}^{1-b} |W_{3}|$$
(13)

$$\frac{(\rho_m)_n}{\rho_0} = \frac{g \,\rho_0 \,L + C_3 \,W_3^{1-b} \,|\, W_3| + C_n \,W_n^{1-b} \,|\, W_n| \left(\frac{\xi_n}{\beta_n} - 1\right)}{g \,\rho_0 \,L + C_n \,\frac{\xi_n}{\beta_n} \,W_n^{1-b} \,|\, W_n|}$$
where  $n = 1, 2$ 
(14)

Using equation (2) and equation (5), the energy equation can be written as,

$$q_n = W_n c_p \frac{2}{\beta_n} \left[ 1 - \frac{(\rho_m)_n}{\rho_0} \right] \quad n = 1, 2$$
(15)  
Replacing  $\frac{(\rho_m)_n}{\rho_0}$  in equation (15) using

equation (14), we can get

$$q_{n} = W_{n} c_{p} \frac{2}{\beta_{n}} \left[ 1 - \frac{g \rho_{0} L + C_{3} W_{3}^{1-b} |W_{3}| + C_{n} W_{n}^{1-b} |W_{n}| \left(\frac{\xi_{n}}{\beta_{n}} - 1\right)}{g \rho_{0} L + C_{n} \frac{\xi_{n}}{\beta_{n}} W_{n}^{1-b} |W_{n}|} \right]$$

$$n = 1, 2$$

$$= C_n W_n^{2-b} |W_n| c_p \frac{2}{\beta} \left[ \frac{1 - \frac{C_3 W_3^{1-b} |W_3|}{C_n W_n^{1-b} |W_n|}}{g \rho_0 L + C_n \frac{\xi_n}{\beta_n} W_n^{1-b} |W_n|} \right]$$

$$n = 1, 2$$

(16)

From continuity equation for a three parallel channel system,  $W_1 + W_2 + W_3 = 0$ or,  $W_3 = -(W_1 + W_2)$ (17)Using equation (16) and (17)

$$\frac{q_1}{q_2} = \frac{C_1}{C_2} \left(\frac{W_1}{W_2}\right)^{2-b} \frac{|W_1|}{|W_2|} \frac{\beta_2}{\beta_1} \left[ \frac{1 - \frac{C_3}{C_1} \left(1 + \frac{W_2}{W_1}\right)^{1-b} \left|1 + \frac{W_2}{W_1}\right|}{1 - \frac{C_3}{C_2} \left(1 + \frac{W_1}{W_2}\right)^{1-b} \left|1 + \frac{W_1}{W_2}\right|} \right] \\ \left[ \frac{g \ \rho_0 \ L + C_2 \ \frac{\xi_2}{\beta_2} W_2^{1-b} \ |W_2|}{g \ \rho_0 \ L + C_1 \ \frac{\xi_1}{\beta_1} W_1^{1-b} \ |W_1|} \right]$$
(11)

Putting b = 1 (laminar flow), Re  $= \frac{DW}{A\mu}$ ,  $\tau = \frac{\xi}{\beta}$ ,

$$r = 2a\left(\frac{\mu_0^2}{\rho_0^2 g D^3}\right), Q = \frac{q \beta D}{2A \mu_0 c_p}$$
 and

 $L_{eff} = 4 m$  in equation (16) we can get

$$Q_n = r_n \operatorname{Re}_n |\operatorname{Re}_n| \frac{1 - \frac{r_3 |\operatorname{Re}_3|}{r_n |\operatorname{Re}_n|}}{1 + \tau_n r_n |\operatorname{Re}_n|} \quad n = 1,2$$
(19)

. .

which is exactly the same as derived by Chato (1963). Hence, using equation (18) the ratio of heat inputs in terms of non-dimensional parameters becomes,

$$R_{q} = \frac{q_{2}}{q_{1}} = \frac{|s|}{s} \left[ \frac{\left\{ \frac{r_{2}}{r_{1}} + \frac{D_{2}r_{2}}{D_{3}r_{1}} \right\} s^{2} + \frac{D_{1}r_{3}}{D_{3}r_{1}} s}{1 + \frac{D_{1}r_{3}}{D_{3}r_{1}} + \frac{D_{2}r_{3}}{D_{3}r_{1}} s} \right]$$
(20)
$$\left[ \frac{\left( \frac{4}{L_{eff}} \right) + \tau_{1}r_{1} |\text{Re}_{1}|}{\left( \frac{4}{L_{eff}} \right) + \tau_{2}r_{2} |\text{Re}_{2}|} \right] \left[ \frac{D_{2}}{D_{1}} \right]$$

where 
$$s = \frac{|\text{Re}_2|}{|\text{Re}_1|}$$

## **Preference number (Y)**

Imagine a situation in which all the heated channels in a parallel channel system are in downflow. As the power is increased, the pressure drop increases and the channels will reverse one by one to upflow. We can say that the channel that reverses first has the strongest "preference for upflow". This is a remarkable way to predict the possibility of flow reversal in a channel of parallel channel systems. But, one of the major weaknesses in it is that one cannot be sure that at what power this will happen. Yahalom and Bein [4] were the first to suggest a downflow preference number. They showed that channel with the highest downflow preference number will flow down first. Similarly, Todreas and Kazimi [5] had given a upflow preference number which showed that the channel with the highest upflow preference number will reverse from downflow to upflow first. Infact, they have suggested that one should calculate the upflow preference number for all the channels in a parallel channel system. From the table of channel number and corresponding preference numbers, one would know that the channel with highest upflow preference number has the strongest preference for upflow. Conversely, channel with lowest upflow preference number is the channel with the highest tendency to reverse to downflow. We have presented the preference numbers given by Yahalom et al. and Todreas et al. in a graphical form in Fig. 2. It can be seen that both the number infers the same thing. That is, when the upflow preference number is low, the downflow preference number is high and vice versa.



Fig. 2: Variation of preference number with nondimensional power ratio

## Effect of increasing high-powered channel diameter

As discussed earlier we have taken a three parallel channel configuration for the analytical analysis. In this analysis Channel 1 is the heated channel with highest heat input and Channel 3 is the adiabatic channel (downcomer). We are varying the power in Channel 2. It can be seen from Fig. 3 that as the heated channel diameter go on increasing, the power ratio at which flow reversal is taking place also increases. That is, the threshold of metastable regime increases with increase in high-powered channel diameter. Subsequently, the zone at which the loop becomes unstable (possibility of multiple steady states) is increasing. This infers that parallel channel natural circulation system destabilize with increase in high-powered channel diameter.



Fig. 3: Effect of high-powered channel diameter on metastable regime

## Effect of increasing low-powered channel diameter

By increasing the channel diameter of lowpowered channel, we get a completely different result. Though the power ratio at which flow reversal takes place increases as in the earlier case, the unstable zone of the system has been reduced. That is, metastable regime reduces with increase in diameter of the low-powered channel. This is shown in Fig. 4.

## Effect of increasing adiabatic channel (downcomer) diameter

The downcomer diameter has a stronger effect on the metastable regime. With increasing the adiabatic channel diameter, the threshold of metastable regime decreases as shown in Fig. 5. The unstable zones also reduce considerably with increase in downcomer diameter. In a nutshell, the downcomer diameter has a stabilizing effect on the parallel channel natural circulation systems.



Fig. 4: Effect of low powered channel diameter on metastable regime



Fig. 5: Effect of adiabatic channel diameter on metastable regime

# Comparative influence of heated channel and adiabatic channel

Fig. 3 shows that the increase in diameter of high-powered channel has a destabilizing effect on the system. Whereas, increase in diameter of the adiabatic channel (downcomer) has got a stabilizing effect on the parallel channel natural circulation systems as seen in Fig. 5. Interestingly, this leaves us wonder which parameter will ultimately decide the fate of stability in the parallel channel loop. Fig. 6 shows that, by increasing the high-powered channel diameter the threshold of metastable regime can be raised asymptotically up to 0.1717. On the other hand, Fig. 7 shows that by decreasing the adiabatic channel diameter the threshold of metastable regime can go up to 1.0.



Fig. 6: Variation of non-dimensional power with high powered channel diameter

This shows that the adiabatic channel diameter has got a stronger influence on the stability behavior of the loop than the heated channels, which reiterates the findings of Chato (1963).



Fig. 7: Variation of non-dimensional power with adiabatic channel diameter



Fig. 8: Effect of loss coefficient on metastable regime

#### Effect of loss coefficient on metastable regime

It is seen from Fig. 8 that with the increase in resistance to flow (loss coefficients), the threshold of metastable regime increases. That is, a high power is required to cause flow reversal for high loss coefficient channel than what is required for low loss coefficient channels.

### Analysis with RELAP5





An analysis was carried out with RELAP5/ Mod 3.2 for a three parallel channel system with a common downcomer. The loop considered for analysis consists of three vertical channels having equal diameters (Inside diameter = 25.5mm, Thickness = 2.7 mm) and equal lengths (5 m) along with common downcomer of same dimensions as that of the channels. These channels are connected to an inlet header (plenum), (Inside diameter = 250 mm, Thickness = 2.7 mm, Length= 964 mm) at the bottom and same size of an outlet header at the top. The channels are heated with equal power input from the outside and the downcomer is kept unheated. Equal amount of heat is assumed to be removed through a tube-in-tube cooler (Inside diameter= 150 mm, Thickness= 2.7 mm) at the top header. The structural material for the loop wall was SS-316 and the volumetric heat capacity of the material used the analysis in was  $3.83 \times 10^6 J/m^3 K$ . The wall roughness was considered to be equal to  $45 \times 10^{-6} m$ . The working fluid was water and the flow state was in turbulent region. The initial conditions were corresponding to ambient pressure (1 bar) and temperature (300 °K) with zero mass flow rates. The channels were divided into 40 numbers of grids after performing a grid independence test. The schematic of the loop and the adopted nodalisation scheme are shown in Fig.9 and Fig. 10 respectively. The various cases studied using RELAP5 is given in Table 1.



Fig. 10: Nodalization scheme adopted in RELAP5

Table 1: Cases studied with initial heat fluxes  $(kW/m^2)$ 

| Case                                  |                   |                        |      |  |  |
|---------------------------------------|-------------------|------------------------|------|--|--|
| No.                                   | Ch-1              | Ch-2                   | Ch-3 |  |  |
| 1                                     | 10                | 10                     | 10   |  |  |
| 2                                     | 5                 | 5                      | 5    |  |  |
| 3                                     | 10                | 0                      | 10   |  |  |
| 4                                     | 5                 | 0                      | 5    |  |  |
| Heat Flux during Transient Conditions |                   |                        |      |  |  |
| Case                                  | Ch-1              | Ch-2                   | Ch-3 |  |  |
| No.                                   |                   |                        |      |  |  |
| 1                                     | 10                | Decrease power till it | 10   |  |  |
|                                       |                   | reverses the flow      |      |  |  |
| 2                                     | 5                 | Decrease power till it | 5    |  |  |
|                                       |                   | reverses the flow      |      |  |  |
| 3                                     | 10                | Increase power till it | 10   |  |  |
|                                       |                   | reverses the flow      |      |  |  |
| 4                                     | 5                 | Increase power till it | 5    |  |  |
|                                       | reverses the flow |                        |      |  |  |

#### Derivation of flow reversal criteria

For a closed loop at steady state, noting that  $\oint \partial p = 0$ , from equation (4) we can get

$$-g\oint\rho\,dz = \sum \left(\frac{fL}{D} + K\right)\frac{W^2}{2\,\rho\,A^2} \tag{21}$$



Now, considering the closed loop formed by Channel 1 and Channel 3 (shown in Fig. I), equation (21) becomes,

$$-g\left[\int_{A}^{B} \rho \, dz + \int_{F}^{E} \rho \, dz\right] = \sum_{A}^{B} \left(\frac{fL}{D} + K\right) \frac{W_{1}^{2}}{2\rho A^{2}}$$

$$+ \sum_{F}^{E} \left(\frac{fL}{D} + K\right) \frac{(W_{1} + W_{2})^{2}}{2\rho A^{2}}$$
(22)

Now, considering the closed loop formed by Channel 2 and Channel 3 (shown in Fig. II) equation (21) becomes,

$$-g\left[\int_{C}^{D} \rho \, dz + \int_{F}^{E} \rho \, dz\right] = \sum_{C}^{D} \left(\frac{fL}{D} + K\right) \frac{W_2^2}{2\rho A^2}$$

$$+ \sum_{F}^{E} \left(\frac{fL}{D} + K\right) \frac{(W_1 + W_2)^2}{2\rho A^2}$$
(23)

Since,  $z_B - z_A = z_D - z_C = z_F - z_E = L$ , equation (22) and (23) becomes

$$\left(\overline{\rho}_{3}-\overline{\rho}_{1}\right)g L = \sum_{Ch-1} \left(\frac{fL}{D}+K\right) \frac{W_{1}^{2}}{2\overline{\rho}_{1}A^{2}} + \sum_{Ch-3} \left(\frac{fL}{D}+K\right) \frac{(W_{1}+W_{2})^{2}}{2\overline{\rho}_{3}A^{2}}$$
(24)

Similarly,

$$\left(\overline{\rho}_{3}-\overline{\rho}_{2}\right)gL = \sum_{Ch-2} \left(\frac{fL}{D}+K\right) \frac{W_{2}^{2}}{2\overline{\rho}_{2}A^{2}}$$

$$+ \sum_{Ch-3} \left(\frac{fL}{D}+K\right) \frac{\left(W_{1}+W_{2}\right)^{2}}{2\overline{\rho}_{3}A^{2}}$$
(25)

The flow in Channel 2 to get reversed, a local loop has to be set up between Channel 1 and 2. This will happen only when the buoyancy force available between Ch-1 and Ch-2 overcomes that of buoyancy force between Ch-2 and Ch-3. That is, for flow reversal from up flow to down flow the condition that should be satisfied is

$$(\overline{\rho}_2 - \overline{\rho}_1)_g L \ge (\overline{\rho}_3 - \overline{\rho}_2)_g L \quad \text{or} (\overline{\rho}_2 - \overline{\rho}_1)_g L \ge (\overline{\rho}_{DC} - \overline{\rho}_2)_g L$$
 (26)

Conversely, the flow reversal criteria from down flow to up flow can be given as

$$(\overline{\rho}_{3} - \overline{\rho}_{2})g L \ge (\overline{\rho}_{2} - \overline{\rho}_{1})g L \quad \text{or} (\overline{\rho}_{DC} - \overline{\rho}_{2})g L \ge (\overline{\rho}_{2} - \overline{\rho}_{1})g L$$
(27)  
At limiting case, just before flow reversal  
$$(\overline{\rho}_{2} - \overline{\rho}_{1})g L = (\overline{\rho}_{3} - \overline{\rho}_{2})g L$$

$$\begin{split} \sum_{Ch-1} & \left(\frac{fL}{D} + K\right) \frac{W_1^2}{2\bar{\rho}_1 A^2} + \sum_{Ch-2} & \left(\frac{fL}{D} + K\right) \frac{W_2^2}{2\bar{\rho}_2 A^2} \\ &= \sum_{Ch-2} & \left(\frac{fL}{D} + K\right) \frac{W_2^2}{2\bar{\rho}_2 A^2} + \sum_{Ch-3} & \left(\frac{fL}{D} + K\right) \frac{(W_1 + W_2)^2}{2\bar{\rho}_3 A^2} \\ &\sum_{Ch-1} & \left(\frac{fL}{D} + K\right) \frac{W_1^2}{2\bar{\rho}_1 A^2} = \sum_{Ch-3} & \left(\frac{fL}{D} + K\right) \frac{(W_1 + W_2)^2}{2\bar{\rho}_3 A^2} \\ & (28) \end{split}$$

Incase of a four parallel channel system that has been analyzed by RELAP5/ Mod 3.2 computer code, the downcomer is equivalent to the Channel 3 of the above mentioned derivation. In order to validate the analytical condition given in equation (26) and (27), the average density difference among the channels have been calculated using RELAP5 before and after flow reversal and given in a tabular form in Table 2.

## Explanation of flow reversal from RELAP5 results

Here Case 1 corresponds to the situation when all three channels were equally heated with a heat flux of 10 kW/m<sup>2</sup> and after steady state, the heat flux in Channel 2 was reduced to cause flow reversal in that channel. Before flow reversal, the average density difference between downcomer

|                         | $\left(\overline{\rho}_{DC}-\overline{\rho}_{1} ight)$ | $\left(\overline{ ho}_{DC}-\overline{ ho}_{2} ight)$ | $(\overline{\rho}_2 - \overline{\rho}_1)$ | $(\overline{\rho}_2 - \overline{\rho}_3)$ | Case No. |
|-------------------------|--|--|---|---|----------|
|                         | in kg/m <sup>3</sup>                                   | in kg/m <sup>3</sup>                                 | in kg/m <sup>3</sup>                      | in kg/m <sup>3</sup>                      |          |
| Before flow<br>reversal | 6.13   | 6.13   | 2.99e-6                                   | 2.44e-6                                   |          |
| After flow<br>reversal  | 3.70   | 0.26   | 3.43                                      | 3.29                                      | Case-1   |
| Before flow<br>reversal | 3.74   | 3.74   | 5.19e-4                                   | 5.19e-4                                   | Case-2   |
| After flow<br>reversal  | 2.31   | 0.15   | 2.15                                      | 2.07                                      |          |
| Before flow<br>reversal | 3.73   | 0.27   | 3.44                                      | 3.29                                      | Case-3   |
| After flow<br>reversal  | 6.16   | 6.16   | 2.23e-6                                   | 2.23e-6                                   |          |
| Before flow<br>reversal | 2.21   | 0.053  | 2.19                                      | 2.17                                      | Case-4   |
| After flow<br>reversal  | 3.19   | 2.81   | 0.35                                      | 0.35                                      |          |

Table 2: Density difference among the channels before and after flow reversal using RELAP5

and Channel 2 (6.13 kg/m<sup>3</sup>) is higher than that between Channel 2 and Channel 1 (2.99e-6  $kg/m^3$ ) (please see Table 2). Clearly this shows before flow reversal the loop is between Ch-2 and downcomer. Whereas after flow reversal the average density difference between Ch-2 and Ch-1  $(3.43 \text{ kg/m}^3)$  has overtaken that between downcomer and Ch-2 (0.26 kg/m<sup>3</sup>). The average density difference between Ch-2 and Ch-3 has also risen considerably, which will in turn promote flow reversal in Ch-2 (up flow to down flow) as the flow in Ch-3 is upward. This is consistent with our analytical condition given in equation (26). Similar explanation can be given for Case 2 where the initial starting heat fluxes in each of the three channels were 5  $kW/m^2$ . Figure 11 shows the variation of average density difference among the channels for Case 2.

Now, Case 3 corresponds to the situation where two channels (Ch-1 & Ch-3) were given an equal heat flux of 10 kW/m<sup>2</sup> and Ch-2 remained unheated up to steady state. After steady state the heat flux in Ch-2 was raised to cause flow reversal in that channel. It can be observed in the table that the flow reversal is taking place when average density difference between the downcomer and Ch-2 overtakes that of Ch-2 and Ch-1. This pave the way to set up a local loop between downcomer and Ch-2 as suggested in equation (27) and in conformity with our theory. Similar explanation can be given for Case 4 where the initial starting heat fluxes were 5  $kW/m^2$ .



Fig. 11: Variation of average density difference among the channels (Case-2)

## Comparison of analytical and RELAP5 result to predict $N_{\rm H}$

Figure 12 shows the variation of nondimensional mass flow rate,  $N_m (=W_{un}/W_h)$  with non-dimensional heat flux ratio,  $N_H$ , for the Case-3. This also compares the result obtained analytically using equation (18) and that obtained using RELAP5 for Case 3.

It should be noted that the objective of the present analysis was to calculate the threshold of metastable regime and to compare it with RELAP5 code. As seen from Fig. 12, it is clear that above point B, the analytical method shows that there is only a unique value of steady state

flow rate. Below point B (corresponding to  $N_{\rm H}$  = 0.24), however, there exists a multiple steady state flow regime with three possible values of flow rate for a given value of heat flux ratio. In the analysis by RELAP5, steady state flow rates were computed starting from a very low value of heat flux with negative flow in the low power channel. The chain dotted curve shows the steady state flow rates predicted by RELAP5 code. At a heat flux ratio of 0.2646, the steady state flow predicted by RELAP5 reverses from negative to positive value, which is close to upper threshold of metastable regime predicted by the analytical model (compare with the N<sub>H</sub> corresponding to point B).



Fig. 12: Comparison of analytical and RELAP5 result

#### Hysteresis effect of loop at steady state

Figure 13 shows both the power level (power rise and power set-back) at which flow reversal occurs. It can be seen that in the case of power rise, the level at which flow reversal takes place is higher than that of power level corresponding to power set back. There is a certain region bounded by a range of heat flux ratios where there are two different steady state points differing in flow direction corresponding to a single heat flux ratio. This special case of static instability is characterized by multiple steady states in different flow directions. The region bounded by this multiple steady state points is known as metastable regime. Welander [7] and Creveling et al. [8] analysed a single channel system where they observed oscillatory behavior of flow before the reversal. In the experiments of Bau et al. [9], this phenomenon was also observed. However, Yahalom et al. [4] and Linzer et al. [10] have not reported this type of oscillatory behavior before flow reversal in parallel channel system in conformity with the present findings.

## Effect of orifice on metastable regime using RELAP5

In a nuclear reactor, the low powered channels are generally provided with orifice to match the outlet condition of all the channels. Hence, in the present analysis an attempt has been made to see the effect of orificing on the metastable regime. The result showed that the power ratio at which flow reversal takes place is slightly elevated using an orifice. Figure 14 shows the variation of power ratio with mass flow rate ratio with and without the use of orifice.



Fig. 13: Hysteresis effect during steady state in parallel channel natural circulation system





#### Effect of initial heat flux on N<sub>H</sub>

Figure 15 shows the variation of nondimensional mass flow rate, N<sub>m</sub>, with nondimensional heat flux ratio, N<sub>H</sub>, for the Case-3 & 4. Initially the heat flux in the unheated channel is zero (i.e.  $q''_{un} = 0$ ). The flow in the unheated channel is downward while the flow in the heated channel is upward (i.e. initially when  $N_{\rm H}=0$ ,  $N_{\rm m}$  is a negative quantity). This state is shown at point A. As soon as we start heating in Ch-2, the flow in this channel decreases in downward direction with N<sub>H</sub>. There is a critical value of N<sub>H</sub> at which the flow reverses. This value is corresponding to point B. There is a jump in curve from B to D, through point C where  $N_m$  is zero. With further increase in  $N_H$ , the value of N<sub>m</sub> increases. It should be noted that



Fig. 15: Effect of initial heat fluxes on  $N_H$ 

the above plot is for particular value of initial heat fluxes and for the case of power increase (=10 W/m<sup>2</sup>s). The curve will be different for different initial value of heat flux and different operating procedure (viz. power rise and power set back). Figure 16 shows the variation of  $(1/N_{\rm H})$  with three different initial heat flux values. It is seen that the ratio  $(1/N_{\rm H})$  decreases with increase in initial heat flux. This is in contrast to the results obtained by Linzer et al. [6] where this value was increasing with increase in initial heat flux value. This may be due to the difference in flow conditions, i.e. ours is a single-phase flow.

#### CONCLUSIONS

The metastable regime in multiple parallel channel system with non-uniform heat inputs has



Fig. 16: Effect of initial heat flux values on  $1/N_{\rm H}$ 

been studied. Steady state behavior of a three channel system with a downcomer, was investigated with RELAP5/MOD3.2 computer code. It is seen that corresponding to certain range of heat flux ratio there are two different steady state points differing in flow direction. If the power is increased in the down-flowing channel keeping the power of the other two channels constant, then it will start flowing upwards only after reaching a critical value of power. Similarly, if the power of an up-flowing channel is decreased, it will start flowing down only after reaching another critical value of power. It was observed that the corresponding two critical values of power are not the same. The region bounded by this two critical value (hysteresis region) has been termed as metastable regime.

A study was also performed to investigate the effect of various parameters like diameter, channel height and orificing at the inlet of heater, on the metastable regime. It was found that the diameter of adiabatic channel has got a strong influence on the metastable regime. The metastable regime increases with increase in diameter of high-powered channel whereas it decreases with increase in diameter of low powered channel. The downcomer diameter has a stabilizing effect on the parallel channel natural circulation systems. The increase in loop height and loss coefficients tends to increase the threshold of metastable regime.

It was found that in a parallel channel natural circulation system with mutually competing driving forces, the actual flow direction is decided by the greater of the two buoyancy forces. The present analysis also showed, with reverse flowing channels considerably large power is required to revert back to upward flow while raising the power for a single-phase system with three channels. A Non-Dimensional Heat Flux Ratio ( $N_H$ ) has been presented to predict the flow reversal. The present analysis also showed that the power level at which flow reversal takes place depends upon the initial power which will be useful in predicting a safe power level for a rational start-up procedure for nuclear reactors. The analysis also showed that the hysteresis regime depends on the rate of reduction or increase of power emphasizing the importance of operating procedure to avoid flow reversal.

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